Mean and Standard deviation:

Example:

Find the average, standard deviation and co-efficient of variation for 821, 783, 834 and 855.

Soln

Mean $\bar{x} = \dfrac{\sum_i x_i}{n}$

$$\therefore \bar{x} = \frac{821 + 783 + 834 + 855}{4}$$

$$= 823.2$$

Standard deviation:

$$s = \sqrt{\sum_i \frac{(x_i - \bar{x})^2}{n-1}}$$

$$S = \sqrt{\frac{(821 - 823.2)^2 + (783 - 823.2)^2 + (834 - 823.2)^2 + (855 - 823.2)^2}{(4-1)}}$$

$$= \sqrt{\frac{(-2.2)^2 + (-40.2)^2 + (10.8)^2 + (31.8)^2}{3}}$$

$$= \sqrt{\frac{4.84 + 1616.04 + 116.64 + 1011.24}{3}}$$

$$= \sqrt{916.25} = 30.3$$

The average (or mean) and standard deviation should both end at the same decimal place.

$\bar{x} = 823.2$

$S = 30.3$

The Coefficient of variation is the Percent relative uncertainty:

$$CV = \frac{S}{\bar{x}} \times 100$$

$$= \frac{30.3}{823.2} \times 100$$

$$CV = 3.7\%$$

4-1. What is the relation between the standard deviation and the precision of a procedure? What is the relation between standard deviation and accuracy?

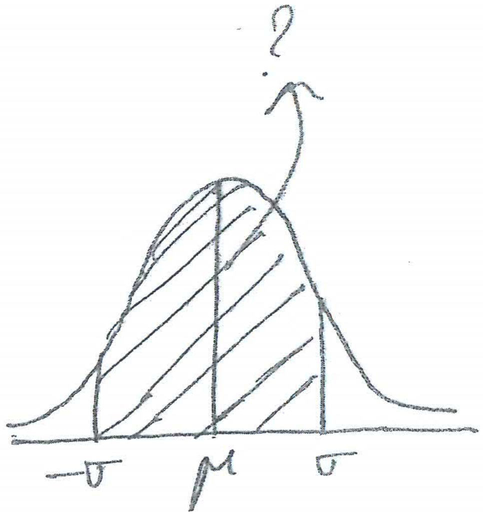
* The smaller the standard deviation, the greater the precision.
* There is no necessary relationship between standard deviation and accuracy.
* The statistics that we do in this chapter pertains to precision, not accuracy.

2. Use Table 4-1 to state what fraction of a Gaussian population lies within the following intervals:

(a) $\mu \pm \sigma$

Soln

$\mu \pm \sigma$

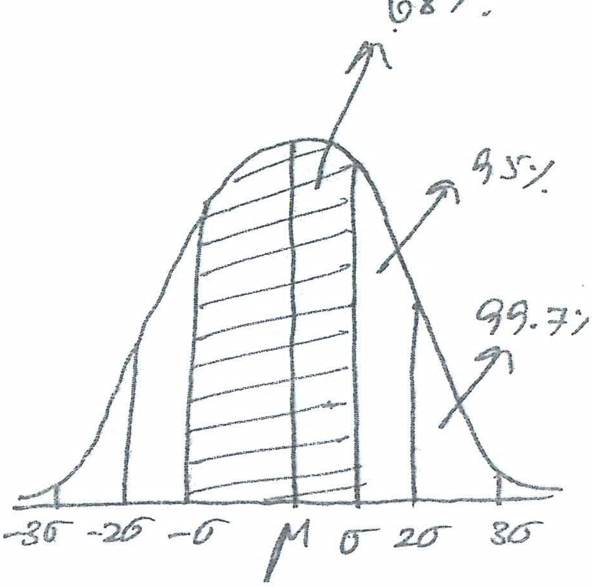


$P(-1 \leq Z \leq +1)$

$= 2P(0 < Z)$

$= 2 \times 0.3413$

$= 0.6826$ Area



$\mu \pm \sigma = 68\%$

$\mu \pm 2\sigma = 95\%$

$\mu \pm 3\sigma = 99.7\%$

$0 \pm 1 \quad (-1, 1)$

$0 \pm 2 \quad (-2, 2)$

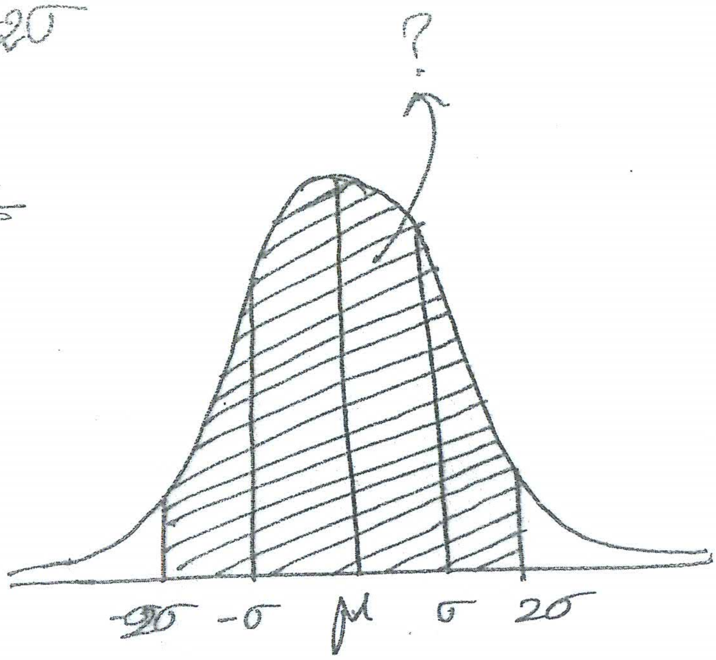
$0 \pm 3 \quad (-3, 3)$

$\mu \pm \sigma$ corresponds to $z = -1$ to $z = +1$.

The area from $z = 0$ to $z = +1$ is 0.3413.

(b) $\mu \pm 2\sigma$

<u>Soln</u>



$\mu \pm 2\sigma$

$P(-2\sigma \leq (x-\mu) \leq 2\sigma)$

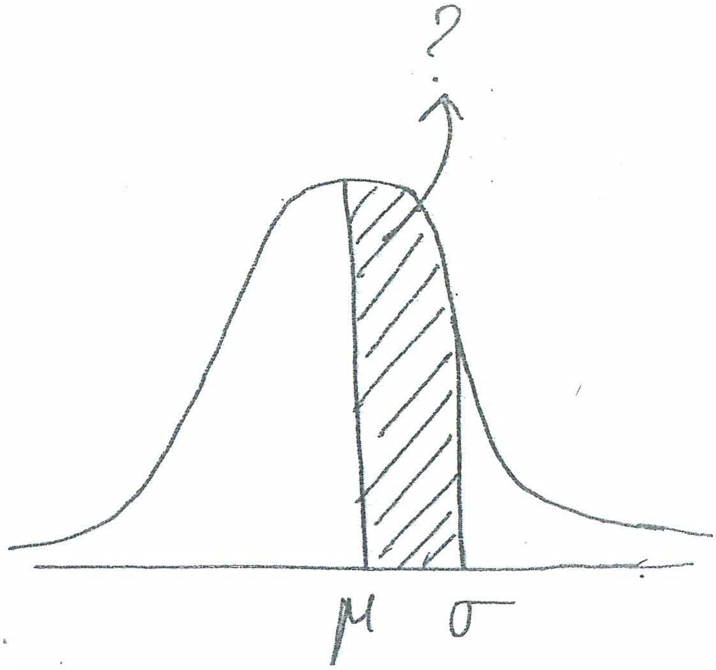
$= P(-2 \leq Z \leq 2)$

$= 2P(0 \leq Z \leq 2)$

$= 2 \times 0.4773$

$Z = 0.9546$ area.

c) μ to $+\sigma$



__Soln__

μ to $+\sigma$

$P(\mu \leq X \leq \sigma)$

$= P(0 \leq Z \leq 1)$

$Z = 0.3413$ Area.

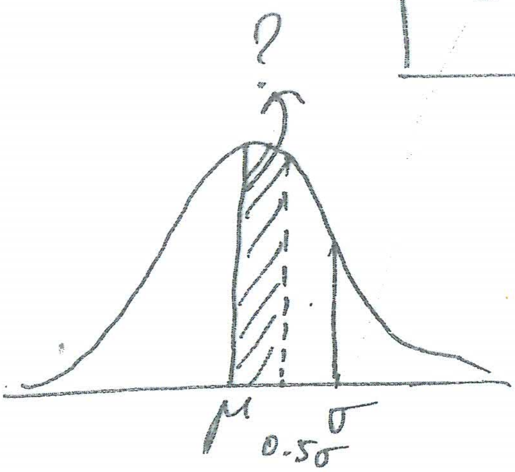
$$Z = \frac{x-\mu}{\sigma}$$

$$Z_1 = \frac{\mu-\mu}{\sigma} = 0$$

$$Z_2 = \frac{\sigma-0}{\sigma} = 1$$

d) μ to $+0.5\sigma$

__Soln__



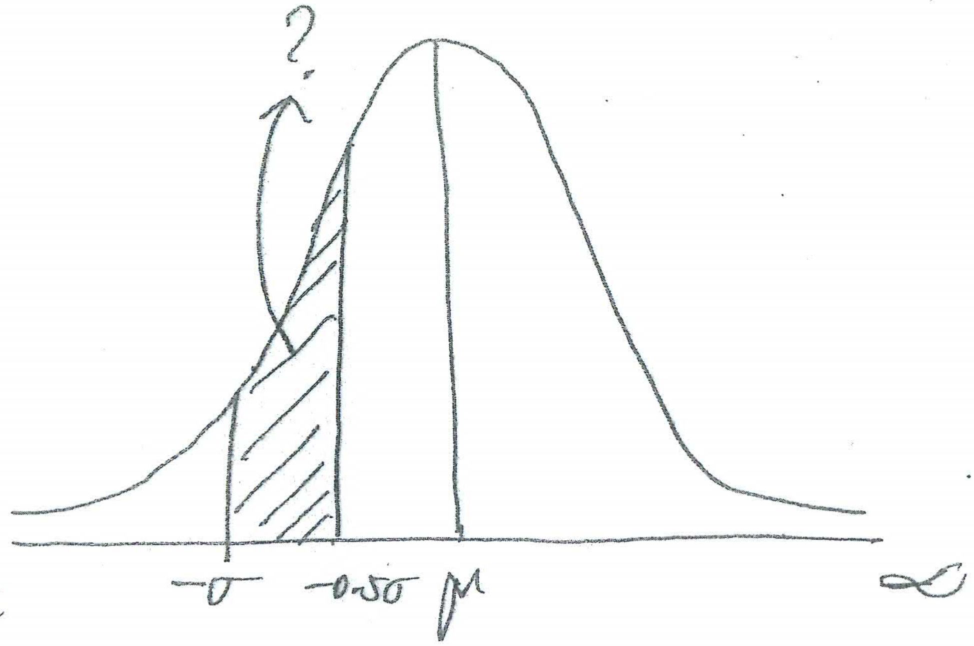
$P(\mu \leq X \leq 0.5\sigma)$

$= P(0 \leq Z \leq 0.5)$

$Z = 0.1915$ area.

) $-\sigma$ to -0.5σ

Soln.



$P(-1\sigma \le X \le -0.5\sigma)$

$P(-1 \le Z \le -0.5)$

Area from

$Z = -1$ to $Z = 0$ is 0.3413

Area from

$Z = -0.5$ to $Z = 0$ is 0.1915

First calculate (0 to 1)

Then calculate (0 to 0.5)

Finally

(0 to 1) − (0 to 0.5)

$\therefore$ Area from $Z = -1$ to $Z = -0.5$ is

$0.3413 - 0.1915 = 0.1498.$

4-3. The ratio of the number of atoms of the isotops ^{69}Ga and ^{71}Ga in eight samples from different sources was measured in an effort to understand differences in reported values of the atomic mass of gallium:

Sample	$^{69}Ga/^{71}Ga$
1.	1.526 60
2.	1.529 74
3.	1.525 92
4.	1.527 31
5.	1.528 94
6.	1.528 04
7.	1.526 85
8.	1.527 93

Soln:

(a) Mean:

$$\bar{x} = \frac{\sum_i x_i}{n}$$

$$= \frac{(1.52660 + 1.52974 + 1.52592 + 1.52731 + 1.52894 + 1.52804 + 1.52685 + 1.52793)}{8}$$

$$= 1.52767$$

(b) Standard deviation:

$$s = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$$

$$s = \sqrt{\frac{(1.52660 - 1.52767)^2 + \dots + (1.52793 - 1.52767)^2}{8-1}}$$

$$s = 0.00126$$

(c) Variance $= S^2$

Variance $= (0.001\ 26)^2$

$= 1.59 \times 10^{-6}$

(d) Significant figures:

$\bar{x} \pm S$

$= 1.527_7 \pm 0.001_3$

(or)

$= 1.528 \pm 0.001$