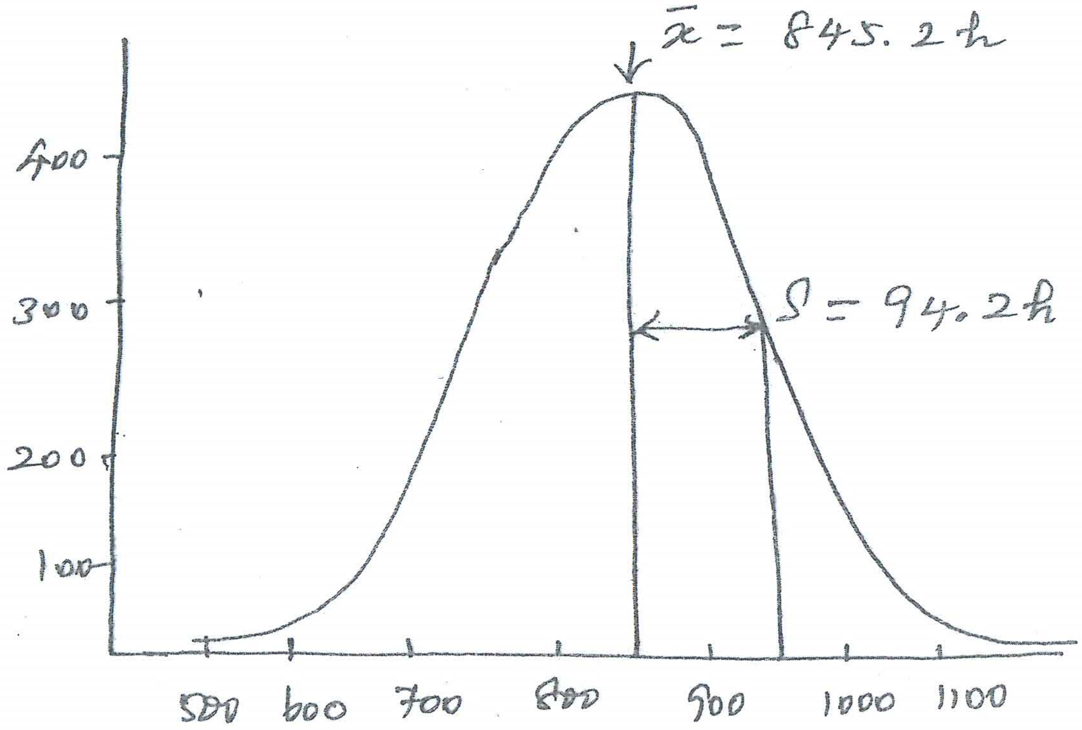
4.4. (a) Calculate the fraction of bulbs in Figure 4-1 expected to have a life time greater than 1005.3h.

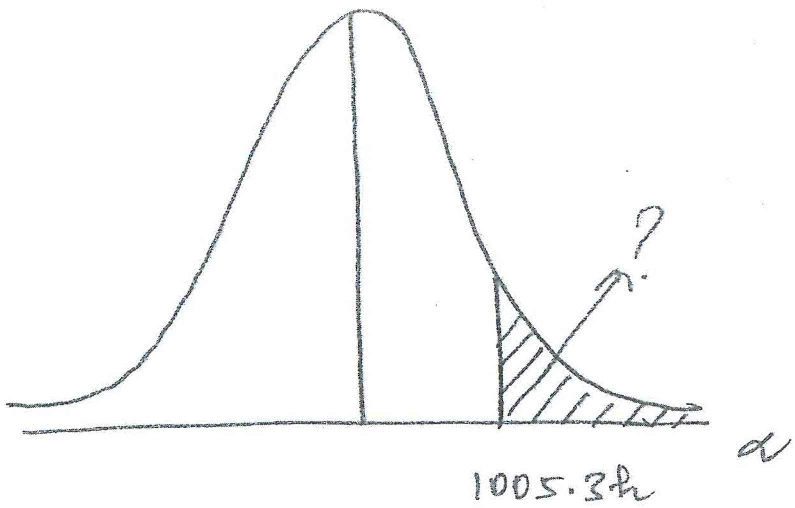


Lifetime (h)
Distribution of x

Soln

$$z = \frac{x-\mu}{\sigma} \approx \frac{x-\bar{x}}{S}$$

z - standardized normal distribution.
x - value of original variation.
μ - population mean.
σ - population standard deviation.



$$Z = \frac{x - \bar{x}}{s}$$

$\because x = 1005.3h$

$\bar{x} = 845.2$

$s = 94.2$

$$Z = \frac{x - \bar{x}}{s} = \frac{1005.3h - 845.2}{94.2}$$

$Z = 1.700.$

$P(x > 1005.3h) = P(Z > 1.700)$

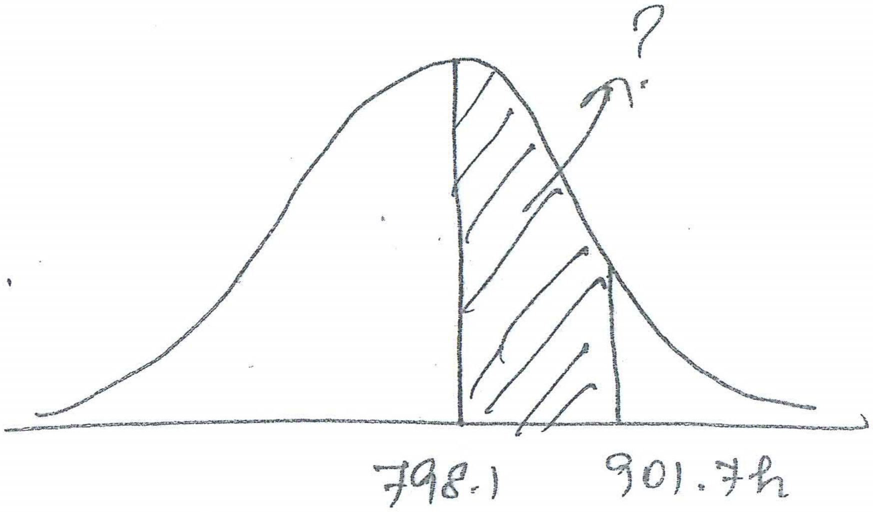
$\therefore$ Area beyond (or) above $Z = 0.500 - P(0 < Z \leq 1.700)$

$= 0.500 - 0.4554$

$= 0.0446$ Area

(In Table 4.1, the area from the mean to $Z = 1.700$ is 0.4554.

(b) What fraction of bulbs is expected to have a life time between 798.1 and 901.7h?



$$Z = \frac{x - \bar{x}}{s}$$

$$Z_1 = \frac{798.1 - 845.2}{94.2}$$

$$= -0.500$$

$$Z_2 = \frac{901.7 - 845.2}{94.2}$$

$$= 0.600$$

$P(798.1 \leq x \leq 901.7h)$

$\therefore P(-0.500 \leq Z \leq 0.600)$

$= P(0 \leq Z \leq 0.500) + P(0 \leq Z \leq 0.600)$

From the table 4.1

$= 0.1915 + 0.2258$

$= 0.4173$

8. what is the meaning of a confidence interval?

A confidence interval is a region around the measured mean in which the true mean is likely to lie.

Confidence Interval:

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

where:

s - is the measured standard deviation.

n - is the number of observation

$\bar{x}$ - is the measured mean

t = is the students t taken from table 4.2.

Example:

Calculating Confidence intervals:

The carbohydrate content of a glyco-protein (a protein with sugars attached to it) is determined to be 12.6, 11.9, 13.0, 12.7 and 12.5 wt% (g carbohydrate/100 g glycoprotein) in replicate analyses. Find the 50% and 90% confidence intervals for the carbohydrate content.

Soln.

Confidence interval $\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$

$$\bar{x} = \frac{12.6 + 11.9 + 13.0 + 12.7 + 12.5}{5}$$

mean $\bar{x} = 12.5_4$

The 90% Confidence interval is

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

$$= 12.5_4 \pm \frac{(2.132)(0.40)}{\sqrt{5}}$$

$$= 12.5_4 \pm 0.3_8$$

The Calculations mean that there is a 50% Chance that the true mean, μ, lies within the range $12.5_4 \pm 0.1_3$ $(12.4_1$ to $12.6_7)$.

There is a 90% Chance that μ lies within the range $12.5_4 \pm 0.3_8$ $(12.1_6$ to $12.9_2)$.

Standard deviation

$$S = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$$

$$S = \sqrt{\frac{(12.6-12.5)^2 + (11.9-12.5)^2 + (13.0-12.5)^2 + (12.7-12.5)^2 + (12.5-12.5)^2}{(5-1)}}$$

$S = 0.4$

50% confidence interval is

$\therefore \mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$

$= 12.54 \pm \frac{(0.741)(0.4)}{\sqrt{5}}$

$= 12.5_4 \pm 0.1_3$

df
$(n-1) = 5$

$\therefore t = 0.741$
from the table.

4-10) List the three different cases that we studied for comparison of means, and write the equations used in each case.

Soln.

Case 1: Comparing a measured Result to a "known" value:

$$t_{calculated} = \frac{|\bar{x} - \text{known value}|}{S}\sqrt{n}$$

Case 2: Comparing Replicate measurements.

$$t_{calculated} = \frac{|\bar{x}_1 - \bar{x}_2|}{S_{pooled}}\sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$S_{pooled} = \sqrt{\frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}}$$

Here S_{pooled} is a pooled standard deviation making use of both sets of data.

$t_{calculated}$ from the equation is compared with t in Table 4-2 for $n_1 + n_2 - 2$ degrees of freedom.

If $t_{calculated}$ is greater than t_{table} at the 95% confidence level, the two results are considered to be different.

ase 3: Comparing individual values:

$$t_{calculated} = \frac{\bar{d}}{S_d}\sqrt{n}$$

$$S_d = \sqrt{\frac{\Sigma (d_i - \bar{d})^2}{n-1}}$$