4-11. The Percentage of an additive in gasoline was measured six times with the following results: 0.13, 0.12, 0.16, 0.17 0.20, 0.11%. Find the 90% and 99% Confidence intervals for the percentage of the additive.

Soln.

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

$$\bar{x} = \frac{0.13 + 0.12 + 0.16 + 0.17 + 0.20 + 0.11}{6}$$

$$\bar{x} = 0.14_8$$

$$s = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$$

Standard deviation $s = 0.03_4$

90% confidence:

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

$$= 0.14_8 \pm \frac{(2.015)(0.034)}{\sqrt{6}}$$

$$= 0.14_8 \pm 0.02_8$$

∵ degree of freedom $(n-1)$
$= 6-1 = 5$

∴ from the t table for 90%

$5 \rightarrow 2.015$

99% confidence:

$$\mu = 0.14_8 \pm \frac{(4.032)(0.034)}{\sqrt{6}}$$

$$= 0.14_8 \pm 0.05_6.$$

∴ $df = (n-1)$
$= 6-1$
$= 5$

From the t-table for 99% confidence

$5 \rightarrow 4.032$

-12) Sample 8 of problem 4-3 was analysed seven times, with $\bar{x} = 1.52793$ and $S = 0.00007$. Find the 99% confidence interval for sample 8.

Soln.

C.I $\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$

$\bar{x} = 1.52793$

$S = 0.00007$

$n = 7$

∵ degrees of freedom $(n-1)$

$= 7 - 1 = 6.$

∴ t values from the table @ 99% C

$6 \Rightarrow 3.707.$

9% Confidence intervals.

$\mu = 1.52793 \pm \frac{(3.707)(0.00007)}{\sqrt{7}}$

$= 1.52793 \pm \frac{2.595 \times 10^{-4}}{\sqrt{7}}$

$= 1.52793 \pm 0.0000980$

$= 1.52793 \pm 0.00010$

[illegible]

23a

4–13.) A trainee in a medical lab will be released to work on her own when her results agree with those of an experienced worker at the 95% confidence level. Results for a blood urea nitrogen analysis are shown below.

Trainee: $\bar{x} = 14.5_7$ mg/dL $s = 0.5_3$ mg/dL $n = 6$ samples

Experienced worker: $\bar{x} = 13.9_5$ mg/dL $s = 0.4_2$ mg/dL $n = 5$ samples

(a) What does the abbreviation dL stand for?

(b) Should the trainee be released to work alone?

23b

(a) dL = deciliter = 0.1L = 100 mL.

(b)
$$F_{Calculated} = \frac{(0.05_3)^2}{(0.04_2)^2}$$

$$= 1.59$$

$$1.59 < 6.26$$

$$F_{Calculated} < F_{Table}.$$

For 5 degree of freedom in the numerator and 4 degrees of freedom in the denominator.

Since $F_{Calculated} < F_{Table}$, we can use the following equations:

$$t_{Calculated} = \frac{|\bar{x}_1 - \bar{x}_2|}{S_{Pooled}} \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$S_{Pooled} = \sqrt{\frac{S_1^2(n_1-1) + S_2^2(n_2-1)}{n_1 + n_2 - 2}}$$

(2b)c

$$S_{pooled} = \sqrt{\frac{0.53^2(5) + 0.42^2(4)}{6+2-2}}$$

$$= 0.48_4$$

$$t = \frac{|14.5_7 - 13.9_5|}{0.48_4}\sqrt{\frac{6\times5}{6+5}}$$

$$= 2.12.$$

$t_{calculated}$ 2.12 < t_{table} 2.262 | listed for 95% confidence and 9 degrees of freedom.

The results agree and the trainee should be released.

4-16) Two methods were used to measure fluorescence lifetime of a dye. Are the standard deviations significantly different? Are the mean significantly different?

Quantity	Method 1	Method 2.
Mean lifetime (ns)	1.382	1.346
standard deviation (ns)	0.025	0.039
Number of measurements	4	4

Soln:

$$F_{calculated} = \frac{S_2^2}{S_1^2}$$

$$= \frac{(0.039)^2}{(0.025)^2}$$

$$= 2.43$$

$\because S_1 = 0.025$

$S_2 = 0.039$

$F_{table} = 9.28$ for 3 degrees of freedom in the numerator and denominator.

Since $F_{calculated} < F_{Table}$,

$2.43 < 9.28$,

the difference in standard deviation is not significant.

we use the Equation

$$S_{pooled} = \sqrt{\frac{S_1^2(n_1-1) + S_2^2(n_2-1)}{n_1+n_2-2}}$$

$$= \sqrt{\frac{(0.025)^2(4-1) + (0.039)^2(4-1)}{4+4-2}}$$

$$= 0.0328$$

$$t_{calculated} = \frac{|\bar{x}_1 - \bar{x}_2|}{S_{pooled}} \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$= \frac{|1.382 - 1.346|}{0.0328} \sqrt{\frac{4 \times 4}{4 + 4}}$$

$$= 1.55$$

t_{table} ($4 + 4 - 2 = 6$ degrees of freedom) $= 2.447$

Since $t_{calculated} < t_{Table}$

$$1.55 < 2.447$$

The difference is not significant.

4-18) If you measure quantity four times and the standard deviation is 1.0% of the average, can you be 90% confident that the true value is within 1.2% of the measured average?

Soln

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

$\bar{x} = \bar{x}$

$s = 1\%$

$n = 4$

From the table t value for

$d.f = n-1$

$= 4-1$

$= 3$

$\therefore \mu = \bar{x} \pm \frac{(2.353)(1\%)}{\sqrt{4}}$

$= \bar{x} \pm 1.18\%$

$= \bar{x} \pm 1.18\% < 1.2\%$

∴ The answer is yes.

∴ t value @ 90% confidence for the d.f = 3

$t = (2.353)$

4-20) Hydrocarbons in the Cab of an automobile were measured during trips on the new Jersey Turnpike and trips through the Lincoln Tunnel connecting new york and new Jersey. The concentrations (± standard deviations) of m-. and p- xylene were.

Turnpike	$31.4 \pm 30.0\ \mu g/m^3$	(32 measurements)
Tunnel	$52.9 \pm 29.8\ \mu g/m^3$	(32 measurements)

Do these results differ at the 95% confidence level? At the 99% confidence level?

Soln:

$$t_{\text{calculated}} = \frac{|\bar{x}_1 - \bar{x}_2|}{S_{\text{pooled}}} \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$S_{\text{pooled}} = \sqrt{\frac{S_1^2 (n_1 - 1) + S_2^2 (n_2 - 1)}{n_1 + n_2 - 2}}$$

$\overline{x}_1 = 31.4$

$S_1 = 30.0$

$n = 32$

$\overline{x}_2 = 52.9$

$S_2 = 29.8$

$n = 32$

$$\therefore S_{pooled} = \sqrt{\frac{S_1^2(n_1-1) + S_2^2(n_2-1)}{n_1+n_2-2}}$$

$$= \sqrt{\frac{(30.0)^2(32-1) + (29.8)^2(32-1)}{32+32-2}}$$

$\therefore S_{pooled} = 29.9.$

$$t_{calculated} = \frac{|\overline{x}_1 - \overline{x}_2|}{S_{pooled}} \sqrt{\frac{n_1 n_2}{n_1+n_2}}$$

$$t_{calculated} = \frac{152.9 - 31.41}{29.9}\sqrt{\frac{32 \times 32}{32+32}}$$

$$= 2.88$$

Degrees of freedom $32+32-2 = 62$

ie $D.F = 62$

In the t_{table} this 62 is close to 60 degrees of freedom.

@ 95% confidence level

$t_{calculated}\,(2.88) > t_{table}\,(2.000)$

@ 99% confidence level

$t_{calculated}\,(2.88) > t_{table}\,(2.660)$

$\therefore$ The difference is significant at the 95 and 99% levels.