Chapter - 4
Statistics.

Example 1:

Find the average and the standard deviation for 821, 783, 834, and 855.

Soln:

Mean $\bar{x} = \dfrac{\sum_i x_i}{n}$

$$\therefore \bar{x} = \frac{821 + 783 + 384 + 855}{4}$$

$$= 823.2$$

Standard deviation:

$$s = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$$

$$s = \sqrt{\frac{(821-823.2)^2 + (783-823.2)^2 + (834-823.2)^2 + (855-823.2)^2}{(4-1)}}$$

$$= \sqrt{\frac{(-2.2)^2 + (-40.2)^2 + (10.8)^2 + (31.8)^2}{3}}$$

$$S = \sqrt{\frac{4.84 + 1616.04 + 116.64 + 1011.24}{3}}$$

$$= \sqrt{916.25}$$

$$= 30.3$$

∴ The average (or mean) and standard deviation should both end at the same decimal place.

$\bar{x} = 823.2$

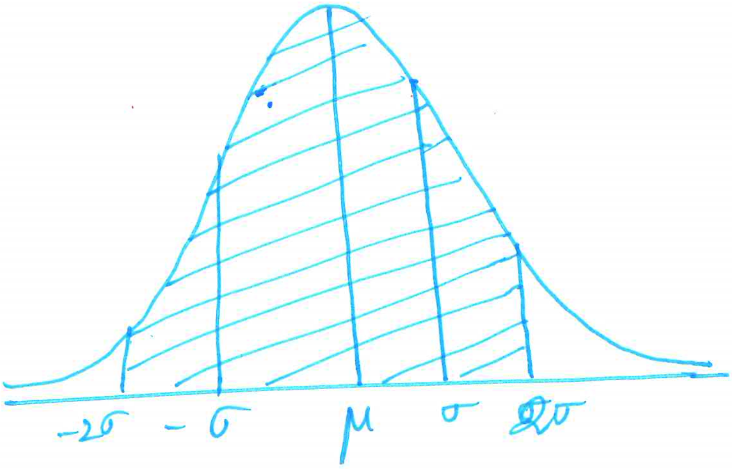
$S = 30.3$

Gaussian Distribution:

A-1. What is the relation between the standard deviation and the precision of a procedure? What is the relation between standard deviation and accuracy?

* The smaller the standard deviation, the greater the precision.
* There is no necessary relationship between standard deviation and accuracy
* The statistics that we do in this chapter pertains to precision, not accuracy.

(b) $\mu \pm 2\sigma$



Soln $\mu \pm 2\sigma$

$$P(-2\sigma \leq (x-\mu) \leq 2\sigma)$$

$$= P(-2 \leq Z \leq 2)$$

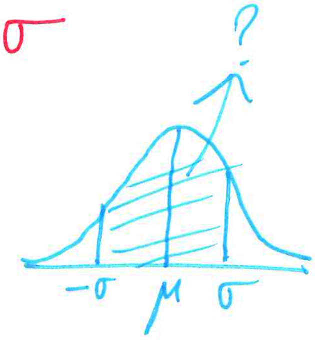
$$= 2P(0 \leq Z \leq 2)$$

$$= 2 \times 0.4773$$

$$= \underline{\underline{0.9546.}} \text{ Area}$$

4-2. use table 4-1 to state what fraction of a Gaussian population lies within the following intervals:

(a) $\mu \pm \sigma$



Soln

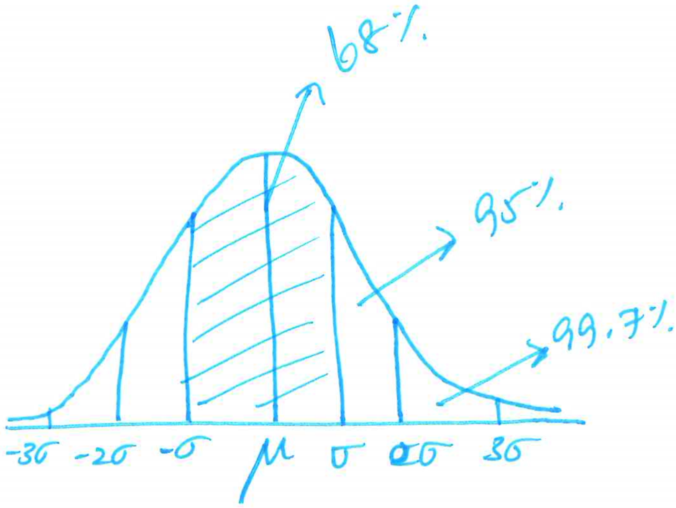
$\mu \pm \sigma$

$P(-1 \le z \le +1)$

$= 2P(0 < z)$

$= 2 \times 0.3413$

$= \underline{\underline{0.6826}}$ Area



$\mu \pm \sigma$ — 68%

$\mu \pm 2\sigma$ — 95%

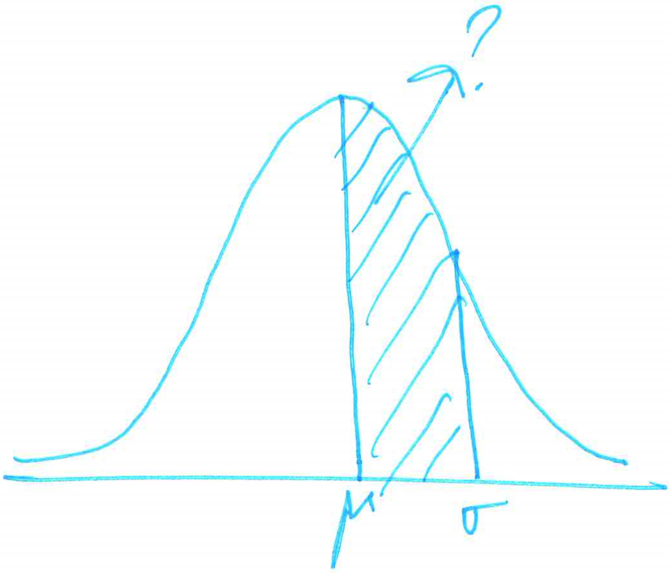
$\mu \pm 3\sigma$ — 99.7%

0 ± 1 $(-1, 1)$

0 ± 2 $(-2, 2)$

0 ± 3 $(-3, 3)$

(c) μ to $\cancel{\pm}\,\sigma$



Soln

μ to $+\sigma$

$P(\mu \le x \le \sigma)$

$= P(0 \le z \le 1)$

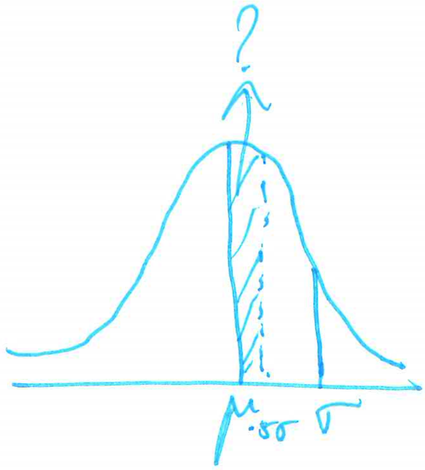
$= \underline{\underline{0.3413}}$ Area

$$Z = \frac{x-\mu}{\sigma}$$

$$Z_1 = \frac{\mu-\mu}{\sigma} = 0$$

$$Z_2 = \frac{\sigma-0}{\sigma} = 1$$

(d) μ to $+0.5\sigma$



Soln

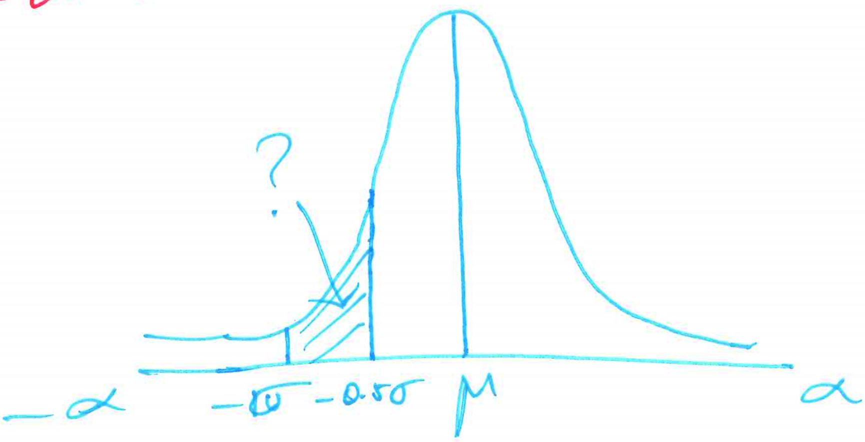
$$P(\mu \le x \le 0.5\sigma)$$

$$= P(0 \le z \le 0.5)$$

$$= \underline{\underline{0.1915}} \text{ Area.}$$

(e) $-\sigma$ to -0.5σ

Soln



$P(-\sigma \le x \le -0.5\sigma)$

$= P(-1 \le Z \le -0.5)$

Area from

$Z = -1$ to $Z = 0$ is 0.3413

Area from

$Z = -0.5$ to $Z = 0$ is 0.1915

$\therefore$ Area from $Z = -1$ to $Z = -0.5$

is

$0.3413 - 0.1915$

Area $= \underline{\underline{0.1498}}$

First Calculate

(0 to 1)

Then Calculate

(0 to 0.5)

Finally

(0 to 1) $-$ (0 to 0.5)

(b) (8)

Example 2:

(4-3) The ratio of the number of atoms of the isotopes ^{69}Ga and ^{71}Ga in eight samples from different sources was measured in an effort to understand differences in reported values of the atomic mass of gallium:

Sample	$^{69}Ga/^{71}Ga$
1.	1.526 60
2.	1.529 74
3.	1.525 92
4.	1.527 31
5.	1.528 94
6.	1.528 04
7.	1.526 85
8.	1.527 93

Find the (a) mean (b) standard deviation (c) variance.

Solution

Sl. no	x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
1.	1.526 60	−0.00107	0.00000114
2.	1.529 74	0.00207	0.0000428
3.	1.525 92	−0.00175	0.00000 306
4.	1.527 31	0.000 36	0.000000129$_6$
5.	1.528 94	0.00127	0.00000 161
6.	1.528 04	0.00082	0.000000 672
7.	1.526 85	0.00082	0.000000672
8.	1.527 93	0.00026	0.0000000676
	$\sum_i x = 12.22133$		1.1563×10^{-5}

$$\text{Mean } \bar{x} = \frac{\sum_i x_i}{n} = \frac{12.22133}{8} = 1.52767$$

8 ~~5~~

$$\therefore S = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{1.156\times10^{-5}}{8-1}} = \sqrt{\frac{1.156\times10^{-5}}{7}}$$

$$= \sqrt{1.6519\times10^{-6}}$$

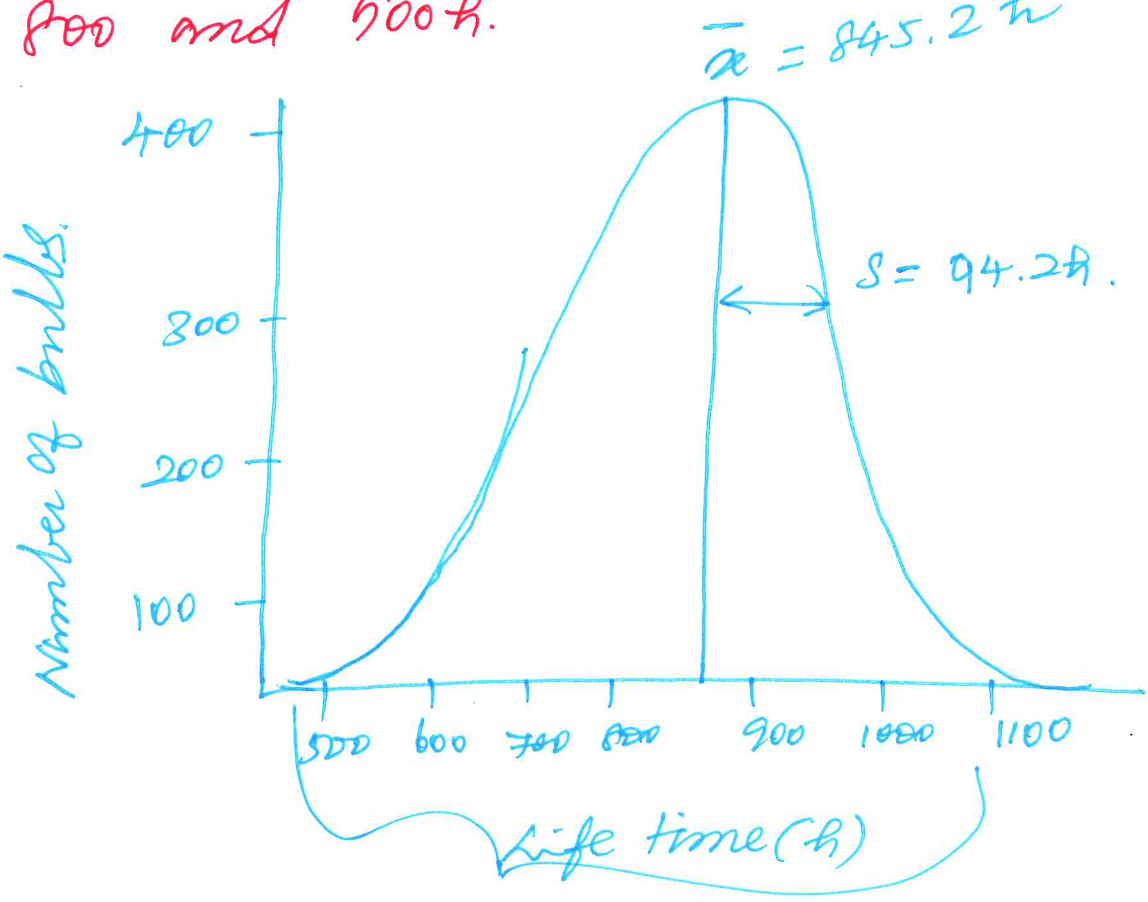
$$S = 0.00128$$

(c) Variance $= S^2$

$$\therefore S^2 = (0.00128)^2$$

$$= 1.64\times10^{-6}$$

4.4 Calculate the fraction of bulbs in Fig. 4-1 expected to have a life time (a) greater than 1000 h and (b) between 800 and 900 h.



Distribution of x

Soln

$$Z = \frac{x - \mu}{\sigma} \simeq \frac{x - \bar{x}}{s}$$

Z – standardised normal distribution.

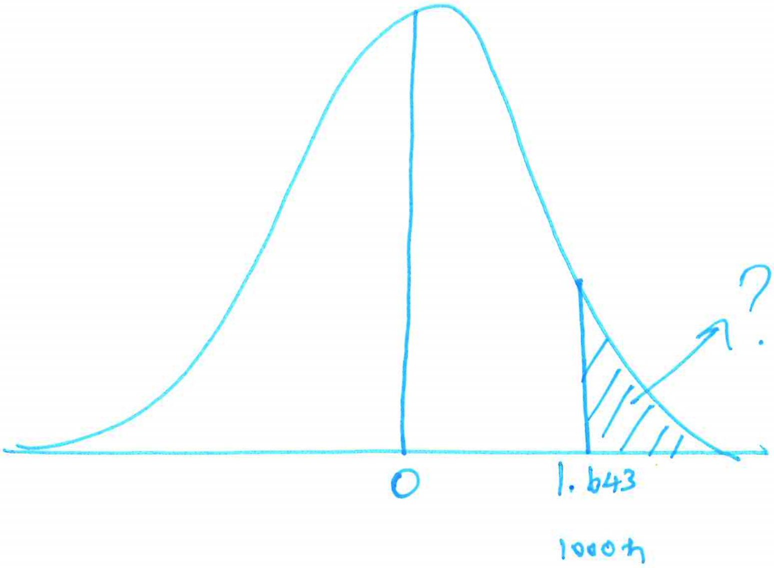
x – value of original variation

μ – population mean

σ – population standard deviation.

(a)

10



$\because x = 1000$

$\bar{x} = 845.2$

$s = 94.2$

$$Z = \frac{x - \bar{x}}{s}$$

$$Z = \frac{x - \bar{x}}{s} = \frac{1000 - 845.2}{94.2}$$

$$Z = 1.643$$

$$P(x > 1000h) = P(Z > 1.643)$$

$\therefore$ Area beyond Z $= 0.5000 - P(0 < Z \leq 1.643)$

$= 0.5000 - 0.4452$

$= \underline{0.0548}$ Area.

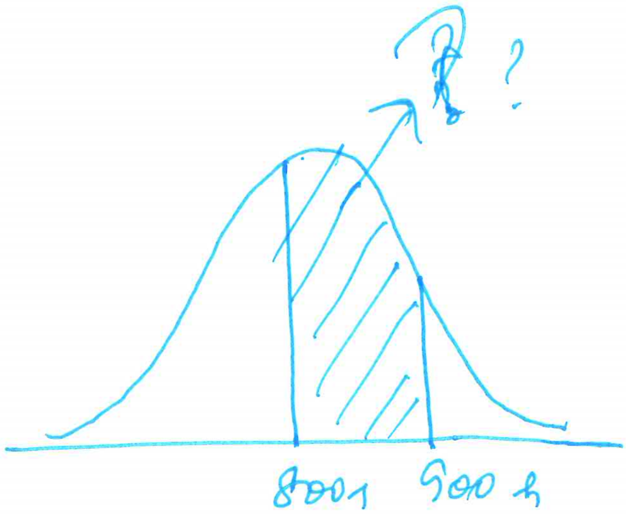
$\therefore$ Area beyond $Z = 1.643$ is

$= 0.5000 - 0.4452$

$= \underline{0.0548}$

(b) between 800 and 900 h.

$$Z = \frac{x - \bar{x}}{s}$$



$P(800 \leq x \leq 900) = ?$

$$Z_1 = \frac{800 - 845.2}{94.2} = -0.479$$

$$Z_2 = \frac{900 - 845.2}{94.2} = 0.582$$

$\therefore P(-0.479 \leq Z \leq 0.582)$

$= P(0 \leq Z \leq 0.479) + P(0 \leq Z \leq 0.582)$

$= 0.1915 + 0.2258$

$= \underline{0.4173}$ Area.

(A.5) Consider a Gaussian distribution with a population mean of 14.49_6 and a population standard deviation of 0.10_7. Find the fraction of measurements expected between 14.55 and 14.60 if many measurements are made.

Soln:-

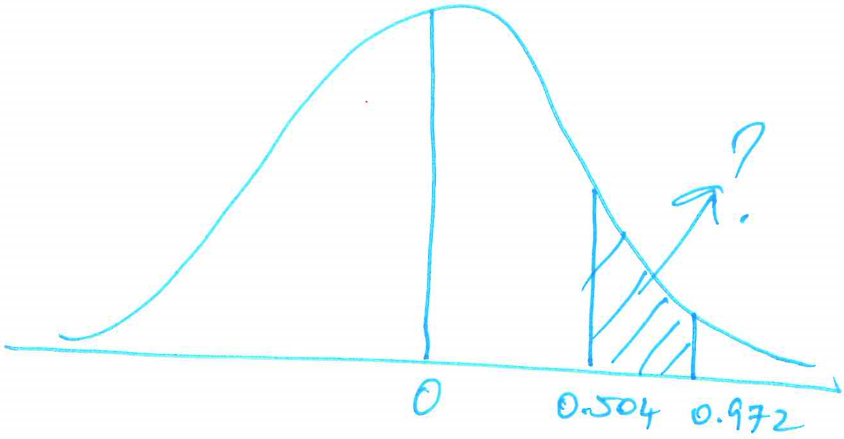
Population mean $\mu = 14.49_6$

population standard deviation $= 0.107$.

$$Z = \frac{x - \bar{x}}{s} \approx \frac{x - \mu}{\sigma}$$

$$Z_1 = \frac{14.55 - 14.496}{0.107} = 0.504_7$$

$$Z_2 = \frac{14.60 - 14.496}{0.107} = 0.972$$



$$\therefore \quad P[14.55 \le x \le 14.60]$$

$$= P[0.504 \le Z \le 0.972]$$

$$= \underset{\sim 1.0}{P[0 \le Z \le 0.972]} - \underset{\sim 0.5}{P[0 \le Z \le 0.504]}$$

$$= 0.3413 - 0.1918$$

$$= \underline{\underline{0.1495}} \text{ Area.}$$

Confidence Intervals:

A confidence interval is a region around the measured mean in which the true mean is likely to lie.

Confidence interval: $\mu = \bar{x} + \frac{ts}{\sqrt{n}}$

where

s – is the measured standard deviation

n – is the number of observations

$\bar{x}$ – is the measured mean.

t = is the students t taken from table A.2.