Example:
calculating confidence intervals:

The carbohydrate content of a glycoprotein (a protein with sugars attached to it) is determined to be 12.6, 11.9, 13.0, 12.7 and 12.5 g of carbohydrate per 100 g of protein in replicate analyses. Find the 50% and 90% confidence intervals for the carbohydrate content.

Soln

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

$$\bar{x} = \frac{12.6 + 11.9 + 13.0 + 12.7 + 12.5}{5}$$

mean $\bar{x} = 12.5_4$

standard deviation $s = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}} =$

(1b)

$$S = \sqrt{\frac{(12.6-12.5)^2+(11.9-12.5)^2+(13.0-12.5)^2+(12.7-12.5)^2+(12.5-12.5)^2}{(5-1)}}$$

df $(n-1) = 4$

$\therefore$ $t = 0.741$ from the table.

$S = 0.4$

$$\therefore \mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

$$= 12.5_4 \pm \frac{(0.741)(0.4)}{\sqrt{5}} = 12.5_4 + 0.1_3$$

The 90% confidence interval is

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

$$= 12.5_4 \pm \frac{(2.132)(0.4_0)}{\sqrt{5}} = 12.5_4 \pm 0.3_8$$

The calculations mean that there is a 50% chance that the true mean, μ, lies within the range $12.5_4 \pm 0.1_3$ (12.4_1 to 12.6_7).

There is a 90% Change that μ lies with in the range 12.54 ± 0.38 (12.16 to 12.92).

(4-10) List the three different cases that we studied for comparison of means, and write the equations used in each case.

Soln

Case 1: Comparing a measured result to a "known" value:

$$t_{calculated} = \frac{|\bar{x} - \text{known value}|}{s}\sqrt{n}$$

Case 2: Comparing replicate measurements.

$$t_{calculated} = \frac{|\bar{x}_1 - \bar{x}_2|}{s_{pooled}}\sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$s_{pooled} = \sqrt{\frac{\sum_{set\,1}(x_i - \bar{x}_1)^2 + \sum_{set\,2}(x_i - \bar{x}_2)^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2}}$$

Here S_{pooled} is a pooled standard deviation making use of both sets of data. $t_{calculated}$ from the ~~ab~~ equation is compared with t in Table 4-2 for n_1+n_2-2 degrees of freedom.

If $t_{calculated}$ is greater than t_{table} at the 95% confidence level, the two results are considered to be different.

Case 3: Comparing individual values:

$$t_{calculated} = \frac{\bar{d}}{S_d}\sqrt{n}$$

$$S_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-1}}$$

4-11 The percentage of an additive in gasoline was measured six times with the following results: 0.13, 0.12, 0.16, 0.17, 0.20, 0.11%. Find the 90% and 99% confidence intervals for the percentage of the additive.

Soln

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

$$\bar{x} = \frac{0.13 + 0.12 + 0.16 + 0.17 + 0.20 + 0.11}{6}$$

$$= 0.14_8$$

$$s = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$$

Standard deviation

$$s = 0.03_4$$

90% confidence :

$$\mu = 0.14_8 \pm \frac{(2.015)(0.03_4)}{\sqrt{6}}$$

$$= 0.14_8 \pm 0.02_8$$

∵ Degree of freedom $(n-1)$
$= 6-1 = 5$

∴ from the t table for 90%

$5 \rightarrow 2.015$

99% confidence :

$$\mu = 0.14_8 \pm \frac{(4.032)(0.03_4)}{\sqrt{6}}$$

$$= 0.14_8 \pm 0.05_6$$

∵ $df = (n-1)$
$= 6-1 = 5$

From the t-table for 99% confidence

$5 \rightarrow 4.032$

$\boxed{A\text{-}12}$ Sample 8 of problem A-3 was analysed seven times, with $\bar{x} = 1.52793$ and $s = 0.00007$. Find the 99% confidence interval for sample 8.

Soln:

C.I $\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$

$\bar{x} = 1.52793$

$s = 0.00007$

$n = 7$

∴ degrees of freedom $(n-1)$

$= 7-1 = 6$

∴ t values from the table @ 99% C

$6 \Rightarrow 3.707$

99% confidence intervals.

$\therefore \mu = 1.52793 \pm \frac{(3.707)(0.00007)}{\sqrt{7}}$

$= 1.52793 \pm \frac{2.595 \times 10^{-4}}{\sqrt{7}}$

$= 1.52793 \pm 0.0000980$

$= 1.52793 \pm 0.00010$

$[1.52783 \text{ to } 1.52803]$

4-17 If you measure quantity four times and the standard deviation is 1.0% of the average, can you be 90% confident that the true value is within 1.2% of the measured average?

Soln

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{n}}$$

From the table t value for

$d.f = n-1$
$= 4-1$
$= 3$

$\therefore$ t value @ 90% Confidence for the $d.f = 3$

$t = (2.353)$

$\bar{x} = \bar{x}$

$s = 1\%$

$n = 4$

$$\therefore \mu = \bar{x} \pm \frac{(2.353)(1\%)}{\sqrt{4}}$$

$$= \bar{x} \pm 1.18\%$$

$$= \bar{x} \pm 1.18\% < 1.2\%$$

The answer is yes.

4.20. A standard reference material is certified to contain 94.6 ppm of an organic contaminant in ~~soil~~. Your analysis gives values of 98.6, 98.4, 97.2, 94.6, and 96.2 ppm. Do your results differ from the expected result at the 95% confidence level? If you made one more measurement and found 94.5, would your conclusion change?

Soln

$x = 98.6, 98.4, 97.2, 94.6$ and 96.2

$n = 5$

$$\therefore \bar{x} = \frac{98.6 + 98.4 + 97.2 + 94.6 + 96.2}{5}$$

$$\therefore \bar{x} = 97.0_0$$

Standard deviation $s = \sqrt{\dfrac{\sum_i (x_i - \bar{x})^2}{n-1}}$

$s = 1.65_5$

$$t_{calculated} = \frac{|\text{Known value} - \bar{x}|}{S} \sqrt{n}$$

$$= \frac{|94.6 - 97.0_0|}{1.6_6} \sqrt{5}$$

$$= \frac{2.4}{1.6_6} \times \sqrt{5}$$

$$= 3.2_3$$

For 4 degrees of freedom and 95% confidence,

$t_{table} = 2.776.$

Here $t_{calculated} (3.23) > t_{table} (2.776)$

∴ the difference is significant.

d.f = $n-1 = 5-1$
$= 4$

t_{value} from the table @ 95%
$= 2.776$

with one more measurement of 94.5

$x = 98.6, 98.4, 97.2, 94.6, 96.2, 94.5$

$n = 6$

$$\therefore \bar{x} = \frac{98.6 + 98.4 + 97.2 + 94.6 + 96.2 + 94.5}{6}$$

$$\therefore \bar{x} = 96.5_8$$

$$S.d \Rightarrow S = 1.8_0$$

$$\therefore t_{calculated} = \frac{|\text{Known value} - \bar{x}|}{S}\sqrt{n}$$

$$= \frac{|94.6 - 96.5_8|}{1.8_0}\sqrt{6}$$

$$= 2.6_9$$

For 5 degrees of freedom and 95% confidence,

$$t_{table} = 2.571.$$

Here $t_{calculated}\ (2.6_9) > t_{table}\ (2.571)$

$\therefore$ The difference is still significant.