Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
2nd Midterm Test
MATH 2202 Linear Algebra I Fall 2010

Monday, December 13, 2010 TIME Allowed: 70 minutes

Attempt all the questions and show all details of your work.

Q 1. [5 marks] Let A, B and C be 3×3 matrices. Let $detA = detB = 2$, $detC = -3$. Evaluate

$$\det\left(C^3 A^{-1} B^T \left(C^T\right)^{-1}\right).$$

Q 2. [7 marks] Let $V = \mathbb{R}^4$ and $H = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \in \mathbb{R}^3 : \begin{array}{c} a + 3b = c \\ b + c + a = d \end{array} \right\}$. Find a matrix A such that $H = NulA$. Then find the vectors that span $NulA$

Q3. [10 marks] Let $A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$. Evaluate $NulA$ and $ColA$.

Q 4. [6 marks] Use Cramer's rule to compute the solution of the following system of equations.

$$\begin{array}{llll} 2x_1 & +x_2 & x_3 & = 4 \\ -x_1 & & +2x_3 & = 2 \\ 3x_1 & +x_2 & +3x_3 & = -2. \end{array}$$

Q 5. [6 marks] Show that the equation of the line in $\mathbb{R}^2$ through distinct points (x_1, y_1) and (x_2, y_2) can be written as

$$det \begin{bmatrix} 1 & x & y \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{bmatrix} = 0.$$

Q 6. [6 marks] Let a and b be positive numbers. Use linear transformation to find the area of the region E bounded by the ellipse whose equation is

$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1.$$

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