

Sultan Qaboos University-College of Science  
Department of Mathematics and Statistics, MATH 2202-Linear Algebra-I  
Midterm-II Examination Spring Semester 2011

Date: 03 May 2011

Time Allowed: 70 minutes

Total Marks 40

ANSWER ALL QUESTIONS ——— SHOW ALL YOUR WORK

---

1. [10 Marks] Let  $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$  and  $b = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$ .
- (i) Find an  $LU$  factorization of the matrix  $A$ .
  - (ii) Use  $LU$  factorization of the matrix  $A$  found in part (i) to solve the equation  $Ax = b$ .
2. [10 Marks] Given the matrix  $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$ .
- (i) Find the determinant of  $A$ .
  - (ii) Find the adjugate (or adjoint) of  $A$ .
  - (iii) Use parts (i) and (ii) to find the inverse of  $A$ .
3. [7 marks] (i) Let  $H = \left\{ \begin{bmatrix} a+3b \\ b-c \\ 2c-a \\ 4b \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$ . Show that  $H$  is subspace of  $\mathbb{R}^4$ .
- (ii) Let  $A = \begin{bmatrix} 1 & -2 & 2 & 3 \\ 3 & -6 & 1 & -1 \\ 2 & -4 & 5 & 8 \end{bmatrix}$  and  $w = \begin{bmatrix} -1 \\ 7 \\ -4 \end{bmatrix}$ . Determine if  $w$  is in  $Col A$ . Is  $w$  in  $Nul A$ ? Justify your answer.
4. [5 Marks] Let  $\mathbf{p}_1(t) = 1 - 3t + 3t^2 - t^3$ ,  $\mathbf{p}_2(t) = 4 - 12t + 9t^2$ ,  $\mathbf{p}_3(t) = 3t^2 - 4t^3$ . Then use **coordinate vectors** to test whether the set of polynomials  $\mathbf{B} = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$  is linearly independent in  $P_3$ .
5. [8 Marks] (i) Let  $A = \begin{bmatrix} 1 & 4 & 5 & -9 \\ -1 & -2 & -1 & 3 \\ 0 & -3 & -6 & 4 \end{bmatrix}$ . Then find the bases of  $Row A$ ,  $Col A$  and  $Nul A$ .
- (ii) If the Null space of a  $9 \times 14$  matrix  $A$  is 8-dimensional, What is the dimension of the space  $Row A$ .
- 

END OF EXAM