

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 - Linear Algebra I
Fall Semester 2008 - Quiz 1 (Version A)

Date: 23rd September 2008

Time Allowed: 20 mins.

NAME

ID NO.

1. [4 marks] The augmented matrix for a system of linear equations is equivalent to the matrix

$$\left[\begin{array}{ccccc} 1 & 2 & -5 & 0 & 5 \\ 0 & 1 & -3 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

List the pivot columns and determine if the system is consistent.

2. [3 marks] Mark the following statement **True** or **False** and justify.
Suppose a 4×6 coefficient matrix of a linear system has four pivot columns. Then, the system is inconsistent.
3. [3 marks] Let $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, show that the vector $\begin{bmatrix} h \\ k \end{bmatrix}$ is in $\text{span}\{\mathbf{u}, \mathbf{v}\}$ for all values of h and k .
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Answers

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 - Linear Algebra I
Fall Semester 2008 - Quiz 1 (Version B)

Date: 23rd September 2008

Time Allowed: 20 mins.

NAME

ID NO.

1. [4 marks] Find the general solution of the system whose augmented matrix is

$$\left[\begin{array}{ccccc|c} 1 & -1 & 2 & -3 & 0 & 0 \\ -3 & 3 & -6 & 5 & 8 & 0 \\ 2 & -2 & 4 & -6 & 0 & 0 \end{array} \right]$$

2. [3 marks] Mark the following statement **True** or **False** and justify.

Suppose a system of linear equations has a 4×6 augmented matrix whose sixth column is a pivot column. Then, the system is consistent.

3. [3 marks] Let $\mathbf{u} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, determine the values of h and k for which $\begin{bmatrix} h \\ k \end{bmatrix}$ is in $\text{span}\{\mathbf{u}, \mathbf{v}\}$.
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Answers

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 - Linear Algebra I
Fall Semester 2008 - Quiz 2 (Version A)

Date: 14th October 2008

Time Allowed: 20 mins.

NAME

ID NO.

1. [4 marks] Mark each statement True or False and Justify.

- (a) If \mathbf{v}_1 and \mathbf{v}_2 are vectors in \mathbb{R}^2 , then $2\mathbf{v}_1 - 3\mathbf{v}_2$ is a linear combination of $3\mathbf{v}_1$ and $2\mathbf{v}_2$.
- (b) Six vectors of \mathbb{R}^5 may form a linearly independent set in \mathbb{R}^5 .

2. [6 marks] Let $A = \begin{bmatrix} -1 & -1 & -2 \\ -2 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

- (a) Do the columns of A span \mathbb{R}^3 ? Justify?
 - (b) Solve the linear system $A\mathbf{x} = \mathbf{b}$. Write the solution in parametric vector form and give a geometric description of the solution set.
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Answers

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 - Linear Algebra I
Fall Semester 2008 - Quiz 2 (Version B)

Date: 14th October 2008
NAME

Time Allowed: 20 mins.
ID NO.

1. [4 marks] Mark each statement True or False and Justify.

- (a) If \mathbf{v}_1 and \mathbf{v}_2 are vectors in \mathbb{R}^2 , then $3\mathbf{v}_1 - 4\mathbf{v}_2$ is a linear combination of $4\mathbf{v}_1$ and $3\mathbf{v}_2$.
- (b) If $\mathbf{v}_1, \dots, \mathbf{v}_4$ are vectors in \mathbb{R}^4 , and $\mathbf{v}_4 = \mathbf{v}_1 + \mathbf{v}_2$, then $\{\mathbf{v}_1, \dots, \mathbf{v}_4\}$ is a linearly independent set in \mathbb{R}^4 .

2. [6 marks] Let $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 2 & 4 & 6 & 8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -1 \\ -7 \\ -4 \end{bmatrix}$

- (a) Do the columns of A span \mathbb{R}^3 ? Justify?
- (b) Solve the linear system $A\mathbf{x} = \mathbf{b}$. Write the solution in parametric vector form and give a geometric description of the solution set.

Answers

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 - Linear Algebra I
Fall Semester 2008 - Quiz 3 (Version A)

Date: 4th November 2008

Time Allowed: 15 mins.

NAME

ID NO.

1. [6 marks] Let $A = \begin{bmatrix} 2 & -2 \\ 1 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ -1 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix}$.

Find the following matrices if they are defined.

(a) $CB - 4I_2$

(b) $A^{-1}B^T$

(c) BC^T

2. [2 marks] Suppose A is an invertible matrix, prove that

$$(A^T)^{-1} = (A^{-1})^T.$$

3. [2 marks] If A is an $n \times n$ matrix, complete the sentences.

(a) A^{-1} exist if and only if the columns of A are.....

(b) A^{-1} exists if and only if $A\mathbf{x} = \mathbf{b}$ has.....

Answers

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 - Linear Algebra I
Fall Semester 2008 - Quiz 3 (Version B)

Date: 4th November 2008

Time Allowed: 15 mins.

NAME

ID NO.

1. [6 marks] Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 0 & 4 \\ -2 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 3 & 2 \end{bmatrix}$.

Find the following matrices if they are defined.

(a) $CB - 4I_2$

(b) $C^T A^{-1}$

(c) $B^T C$

2. [2 marks] Suppose A and B are two invertible $n \times n$ matrices, prove that

$$(AB)^{-1} = B^{-1}A^{-1}.$$

3. [2 marks] If A is an $n \times n$ matrix, complete the sentences.

(a) A^{-1} exists if and only if A has.....

(b) A^{-1} exists if and only if $A\mathbf{x} = \mathbf{0}$

Answers

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 - Linear Algebra I
Fall Semester 2008 - Quiz 4 (Version A)

Date: 18th November 2008

Time Allowed: 20 mins.

NAME

ID NO.

1. [3 marks] Let A be a square matrix. Mark each statement True or False. Justify your answer.

(a) If $\det A$ is zero, then one row or one column of A is zero.

(b) $\det(A^T A) \leq 0$.

2. [2 marks] Find the inverse of the block matrix $\begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$.

3. [2 marks] Use row eliminations to show that $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$

4. Let A and B be 3×3 matrices with $\det A = -3$, and $\det B = \pi$. Then,

(a) $\det(BA^T A) =$

(b) $\det(3B^2) =$

Answers

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 - Linear Algebra I
Fall Semester 2008 - Quiz 4 (Version B)

Date: 18th November 2008
NAME

Time Allowed: 20 mins.
ID NO.

1. [3 marks] Let A be a square matrix. Mark each statement True or False. Justify your answer.
 - (a) If $\det A$ is zero, then two rows or two columns of A are the same.
 - (b) $\det(AA^T) \geq 0$.
2. [2 marks] Find the inverse of the block matrix $\begin{bmatrix} I & A \\ 0 & I \end{bmatrix}$.
3. [2 marks] Use row eliminations to show that $\begin{vmatrix} 7 & 8 & 9 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0$
4. Let A and B be 3×3 matrices with $\det A = e$, and $\det B = -2$. Then,
 - (a) $\det(ABA^T) =$
 - (b) $\det(2B^2) =$

Answers

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 - Linear Algebra I
Fall Semester 2008 - Quiz 5 (Version A)

Date: 21st December 2008

Time Allowed: 20 mins.

NAME

ID NO.

1. [4 marks] Show that the set $\mathfrak{B} = \{1, 1 - t, 2 - 4t + t^2\}$ is a basis for \mathbb{P}_2 (set of polynomials of degree at most 2 with real coefficients).
 2. [3 marks] For $p(t) = 7 - 8t + 3t^2$, find the coordinate vector of $p(t)$ relative to the basis \mathfrak{B} given in question 1.
 3. [3 marks] Mark each statement true or false. Justify your answer.
 - (a) If B is an echelon form of a matrix A , then the pivot columns of B form a basis for $\text{Col } A$.
 - (b) If $\dim V = n$ and S is a linearly independent set in V , then S is a basis for V .
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Answers

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 - Linear Algebra I
Fall Semester 2008 - Quiz 5 (Version B)

Date: 21st December 2008

Time Allowed: 20 mins.

NAME

ID NO.

1. [4 marks] Show that the set $\mathfrak{B} = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$ is a basis for \mathbb{P}_2 (set of polynomials of degree at most 2 with real coefficients).
2. [3 marks] For $p(t) = 3 + t - 6t^2$, find the coordinate vector of $p(t)$ relative to the basis \mathfrak{B} given in question 1.
3. [3 marks] Mark each statement true or false. Justify your answer.
 - (a) The columns of an invertible $n \times n$ matrix A form a basis for $Col A$.
 - (b) If $\dim V = n$ and S spans V , then S is a basis for V .