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Department of Mathematics and Statistics
Linear Algebra I
Test I
Spring 2010

Monday, 22 March 2010
Time Allowed: 70 minutes
Total Mark: 40

(1) Let $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & \frac{3}{2} & -3 \\ 0 & 1 & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ -4 \\ -4 \end{bmatrix}$ and $\mathbf{p} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

(a) [4 marks] Show that A is row equivalent to B.
(b) [2 marks] Do the columns of A span $\mathbb{R}^4$? Justify your answer.
(c) [3 marks] Solve the linear system $A\mathbf{x} = \mathbf{0}$. Write the solution in a parametric form.
(d) [3 marks] Verify that $A\mathbf{p} = \mathbf{b}$. Then, obtain the general solution of the linear system $A\mathbf{x} = \mathbf{b}$.

(2) (a) [6 marks]
Given the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Determine whether the following set of vectors are linearly independent. Justify your answer.
(i) $\{\mathbf{v}_1, \mathbf{v}_4\}$, (ii) $\{\mathbf{v}_3, \mathbf{v}_4, -2\mathbf{v}_3 + 5\mathbf{v}_4\}$, (iii) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.

(b) [4 marks]
Let $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in span $\{\mathbf{u}, \mathbf{v}\}$ for all h and k.

(3) Let $T : \mathbb{R}^2 \to \mathbb{R}^4$ be a linear transformation given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \\ -x_1 + x_2 \\ -x_1 - x_2 \end{bmatrix}$.

(a) [2 marks] Write down the standard matrix A of T.

(b) [2 marks] Is T one to one? Justify your answer.

(c) [2 marks] Is T onto $\mathbb{R}^4$? Justify your answer.

(d) [2 marks] Determine if $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ is in the image of the transformation T.

(4) (a)[5 marks] Find the inverse of $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$ if it exists.

(b) [5 marks] Prove that any linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ can be represnted by an mxn matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for each $\mathbf{x}$ in $\mathbb{R}^n$.