

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 2202 Linear Algebra I Fall 2010

Monday, November 1, 2010

TIME Allowed: 70 minutes

Attempt all the questions and show all details of your work.

Question 1. [5 marks] For what values of h is $v_3 \in \text{span}\{v_1, v_2\}$? and for what values of h is $\{v_1, v_2, v_3\}$ linearly dependent? Justify your answers.

$$v_1 = \begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -9 \\ h \end{bmatrix}.$$

Question 2. [5 marks] Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation from \mathbb{R}^3 into \mathbb{R}^2 . Let $T\left(\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Calculate $T\left(\begin{bmatrix} 11 \\ -13 \\ 6 \end{bmatrix}\right)$.

Question 3. [4 marks] Use the determinant to determine the value(s) of a such that the set $\left\{\begin{bmatrix} 1 \\ a \end{bmatrix}, \begin{bmatrix} a \\ a+2 \end{bmatrix}\right\}$ is linearly independent.

Question 4. [4 marks] Give an example of 2×2 non-zero matrices A , B and C such that $AB = 0$ and $AB = AC$ but $B \neq C$. Explain your example.

Question 5. [8 marks] Suppose A , B and X are $n \times n$ matrices with A , X and $A - AX$ invertible. Suppose that $(A - AX)^{-1} = X^{-1}B$. Show that B is invertible and then solve for X .

Question 6. [14 marks] Show that the following statements are true. Justify your answer.

(a) Let A and B be two matrices such that AB is defined. If the columns of AB are linearly independent then the columns of A are linearly independent.

(b) If A is an $n \times n$ invertible matrix, then A^{-1} is unique.

(c) The linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}$ defined by $T(x_1, x_2, x_3, x_4) = 2x_1 + 3x_3 - 4x_4$ is not one to one but onto.

————— // —————