SULTAN QABOOS UNIVERSITY - COLLEGE OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 2202- Linear Algebra I

Date: 15.12.2009	**MIDTERM EXAM 2 - Fall 2009**	Time: 60 minutes

Answer the following questions (show all work).

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1. (a) [10 marks] Evaluate $\det A$, $\det(-A)$, and $\det(A^T)^2A^{-1}$, where

$$A = \begin{bmatrix} 2 & 4 & -2 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 3 & 1 & 1 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

   (b) Use Cramer's rule to find the $4th$ component of the $4th$ column of A^{-1}.

2. [10 marks] Let $A = \begin{bmatrix} 2 & -2 & 4 \\ 1 & -3 & 1 \\ 3 & 7 & 5 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

   (a) Find an LU factorization of the matrix A. Check your answer.

   (b) Use the LU factorization found in (a) to solve the equation $A\mathbf{x} = \mathbf{b}$.

   (a) Suppose A_{22} is invertible. Find X, and Y such that
$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} I & X \\ 0 & I \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} I & 0 \\ Y & I \end{bmatrix}$,
where $S = A_{11} - A_{12}A_{22}^{-1}A_{21}$.

   (b) Let $A = UDV^T$, where U,V are $n \times n$ matrices such that $U^TU = I$, $V^TV = I$, and D is a diagonal matrix with positive entries $\sigma_1, \sigma_2, ..., \sigma_n$ on the diagonal. Show that A is invertible and find a formula for A^{-1}.

3. [10 marks] The following statements are true. Explain why?

   (a) Let A be an invertible$n \times n$ matrix, and $E_1, E_2, ..., E_p$ are elementary row replacement matrices that transform A into I, then they also transform I into A^{-1}.

   (b) If A is an $n \times n$ matrix such that: $A^TA = I$, then $\det A = \pm 1$.

   (c) Let A be an $n \times n$ matrix such that: $A^2 - 8A + 2I = 0$. Show that A is invertible and that $A^{-1} = -\frac{1}{2}A + 4I$.

Total: 40 marks