SULTAN QABOOS UNIVERSITY - COLLEGE OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 2202- Linear Algebra I

Date: 2.11.2009	**MIDTERM EXAM 1 - FALL 2009**	Time: 60 minutes

Answer the following questions (show all work).

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1. [15 marks] Let $A = \begin{bmatrix} 1 & -4 & 7 & -5 \\ 0 & -1 & 4 & -3 \\ -2 & 6 & -6 & 4 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & 0 & -9 & 7 \\ 0 & 1 & -4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, and $b = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$.

   (a) Show that A is row equivalent to B.

   (b) Solve the linear system $A\mathbf{x} = \mathbf{0}$. Write the solution in parametric vector form, and give a geometric description of the solution set.

   (c) Are the columns of A linearly independent? Justify your answer.

   (d) Do the columns of A span $\mathbb{R}^3$? Justify your answer.

   (e) Is $\mathbf{b}$ in the range of the matrix transformation $T : \mathbf{x} \to A\mathbf{x}$? Justify your answer.

2. [9 marks] Determine the value(s) of the parameters a and b so that the system

$$\begin{cases} x_1 & + & x_2 & + & x_3 & = & 1 \\ ax_1 & & & + & x_3 & = & 2b \\ & + & 3ax_2 & + & x_3 & = & 4 \end{cases} \; ; \text{ has}$$

   (a) no solution ; (b) a unique solution ; (c) infinitly many solutions

3. [7 marks]

   (a) Given $\mathbf{v} \neq \mathbf{0}$ and $\mathbf{p}$ in $\mathbb{R}^n$, show that a linear transformation maps the line through $\mathbf{p}$ in the direction of $\mathbf{v}$ into another line or a single point.

   (b) If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation that maps $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ into $\begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ into $\begin{bmatrix} 6 \\ -5 \\ -3 \end{bmatrix}$. Find the image under T of $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

4. [9 marks] The following statements are **true**. Give a justification for each one.

   (a) If $A\mathbf{x} = \mathbf{b}$ has a solution and $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then the solution of $A\mathbf{x} = \mathbf{b}$ is unique.

   (b) If $\mathbf{v}_{1,}\mathbf{v}_{2,}\mathbf{v}_{3,}$.and $\mathbf{v}_4$ are linearly independent vectors in $\mathbb{R}^4$, then the set $\{\mathbf{v}_{1,}\mathbf{v}_{2,}\mathbf{v}_3\}$ is linearly independent.

   (c) The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by: $T(x_1, x_2) = (1 + x_1^2, x_1 + x_2)$ is not linear.

Total: 40 marks