

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 1106-Precalculus-Test1-Fall 2004

Date: 13/10/2004

Total Marks: 40

Time: 1 hour

Answer all questions and show all your work to get full credit.

1. (a) [4 marks] Solve the inequality and express the answer in interval form:

$$\frac{2x^2 + 7}{3x + 1} > x - 1.$$

- (b) [5 marks] Solve the equation: $x^2 - |4 - 3x| = 0$.

2. A circle has the equation: $x^2 + y^2 - 2x + 6y - 1 = 0$.

- (a) [5 marks] Find an equation of the line passing through the center of the circle and perpendicular to the line $x - 2y - 1 = 0$.

- (b) [3 marks] Find the domain of f defined by: $f(x) = \sqrt{\frac{2 - |x|}{|x - 2|}}$.

3. (a) [6 marks] Starting from the graph of $f(x) = \sqrt{x}$, use transformations to sketch step by step the graph of $g(x) = \frac{1}{2} - \sqrt{1 - x}$, find all intercepts and show them clearly on the graph.

- (b) [2 marks] State the range of g .

4. Let $f(x) = -x^2 + 4x + 5$.

- (a) [2 marks] Write f in the form $f(x) = a(x - h)^2 + k$, where a , h , and k are constants.

- (b) [3 marks] Sketch the graph of f , find all intercepts, and show them clearly on the graph.

- (c) [4 marks] Find the range of f , the interval where the function is increasing, and the interval where it is decreasing.

5. The sum of the width x and the length y of a rectangle is 100 cm.

- (a) [4 marks] Find a function that models the length D of the diagonal of the rectangle in terms of x .

- (b) [2 marks] Find the value of x and of y for which the square of the length D of the diagonal is minimized.

DO YOUR BEST!

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 1106-Precalculus-Test2-Fall 2004

Date: 1/12/2004

Total Marks: 40

Time: 1 hour

Answer all questions and show all your work to get full credit.

Read all questions first and then you may start from any question.

1. Solve the following equations for x :

(a) [4 marks] $\frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1}{2}$

(b) [4 marks] $\{\text{Log}(x)\}^3 = \text{Log}(x^9)$

2. (a) [3 marks] Let $Q(x) = (x^2 - 2x + 1)(x^2 - 4x + 5)$. Factor completely $Q(x)$ into linear factors.

(b) [4 marks] Let $P(x) = x^4 - 6x^3 + 14x^2 - 14x + 5$. Find all real and complex zeros for $P(x)$.

3. (a) [4 marks] Starting from the graph of $f(x) = 2^x$, use transformations to sketch step by step the graph of $g(x) = 4 - 2^{x-1}$, find all intercepts and show them clearly on the final graph.

(b) [3 marks] State the domain, range, and the horizontal asymptote of g .

(c) [5 marks] Show that g is a one-to-one function. Find g^{-1} and its domain.

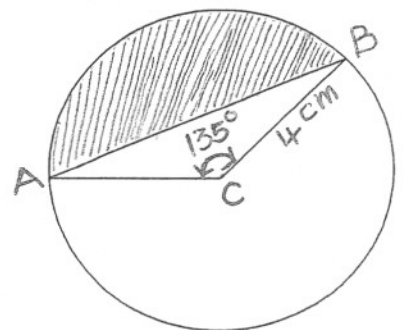
4. Let \widehat{AB} be an arc that subtends a central angle of measure 135° in a circle of radius 4 cm as depicted in the figure below:

(a) [2 marks] Find the reference angle for $\theta = 135^\circ$.

(b) [2 marks] Find the area of the triangle ABC

(c) [2 marks] Find the area of the shaded sector.

(d) [3 marks] Find the length of the arc \widehat{AB} that subtends the angle of 135° .



5. A flagpole of length L is placed on the top of a tower of height 250 ft. From a point D at a distance of 500 ft away from the base of the tower, it is observed that the angle of elevation to the top of the flagpole is 30° .

(a) [2 marks] Find the length L of the flagpole.

(b) [2 marks] Find the distance from the point D to the top of the flagpole.

Do your Best now, use the facilities of the Precalculus Help Center after this Test, then you will be performing much better in the Final Exam.

Sultan Qaboos University-College of Science
Department of Mathematics and Statistics
MATH 1106-Precalculus-FINAL EXAM-Fall 2004

Date: 18/12/2004

Total Marks: 90

Time: 2h30mn

Answer all 10 questions and show all your work to get full credit

1. Solve for x : (a) [5 marks] $9^x + 3^{x+1} - 4 = 0$. (b) [5 marks] $\ln(x) + \ln(x - 3) = 2 \ln(2)$.
2. (a) [3 marks] Write $z = -1 - i$ in trigonometric form.
(b) [6 marks] Find the cubic roots of z .
(c) [4 marks] Sketch the set $\{z = a + bi ; |a| \leq 1 \text{ and } |b| \leq 2\}$ in the complex plane.
3. [6 marks] Express $-\sqrt{3} \sin(x) + \cos(x)$ in the form $k \sin(x + \phi)$.
4. (a) [3 marks] Show that $\cos(2x) = 1 - 2 \sin^2(x)$.
(b) [8 marks] Find all solutions of the equation $\cos(2x) + \sin(x) = 0$.
5. [7 marks] Use the formula for $\sin(u - v)$ to rewrite $\sin(\tan^{-1}(x) - \sin^{-1}(x))$ as an algebraic expression in x .
6. [6 marks] Find all intercepts and asymptotes of the function $f(x) = \frac{x(x+2)}{(x-3)(x+3)}$.
7. (a) [5 marks] Find the domain of $r(x) = \frac{\ln(1-x)}{\ln(1+x)}$.
(b) [5 marks] Find $f \circ g$ and its domain, where $f(x) = \sqrt{x}$ and $g(x) = \frac{x}{x^2 - 1}$.
8. (a) [3 marks] Find the amplitude, phase shift, and the period of $h(x) = 2 \sin(2x - 2\pi)$.
(b) [5 marks] Sketch the graph of one complete period of h showing clearly all x -intercepts.
(c) [3 marks] Find the value of x for which $h(x)$ has a maximal value.
9. [8 marks] If the initial size of a population is n_0 and the population grows exponentially with relative growth rate r , then the expression for the population $n(t)$ at time t is $n(t) = n_0 e^{rt}$.
A bacteria culture starts with 500 bacteria and after 3 hours there are 8000 bacteria.
(a) Find the function that models the number of bacteria after t hours.
(b) Find the number of hours required for the number of bacteria to double.
(c) Find the number of bacteria after 4 hours.
10. [8 marks] Prove that: $\tan(3x) = \frac{3 \tan(x) - \tan^3(x)}{1 - 3 \tan^2(x)}$. (Help: $\tan(a+b) = \frac{\tan(a) + \tan(b)}{1 - \tan(a)\tan(b)}$)

END OF THE EXAM.