

M1106 - Test 1 - Fall 2009

Part I: Multiple Choice Questions

• Each Multiple Choice Question is worth 2 marks for a total of 14 marks out of 50.

1. Simplify $\left(4 - \frac{5}{x}\right) \div \left(5 + \frac{3}{x}\right)$
(A) $\frac{4-5x}{5+3x}$ (B) $\frac{5x+3}{4x-5}$ (C) $\frac{4x^2-5}{5x^2+3}$ (D) $\frac{4x-5}{5x+3}$
2. Rationalize the numerator $\frac{\sqrt{6}-\sqrt{3}}{3}$
(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{6}+\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{3}{\sqrt{6}-\sqrt{3}}$
3. Which of the following lines is perpendicular to $2x - 3y + 5 = 0$:
(A) $2x + 3y - 1 = 0$ (B) $3x - 2y + 5 = 0$ (C) $-4x + 6y - 5 = 0$ (D) $6x + 4y - 1 = 0$
4. If the graph of $f(x) = \sqrt{x}$ is shifted to the left 2 units, reflected in the x -axis, and shifted upward 3 units, then the equation of the final transformed graph is:
(A) $3 - \sqrt{x-2}$ (B) $2 - \sqrt{x-3}$ (C) $3 - \sqrt{x+2}$ (D) $2 - \sqrt{x+3}$
5. The graph of the equation $x^2y^4 - 3xy^2 = 2$ is symmetric about the:
(A) x -axis (B) y -axis (C) the origin (D) None
6. The center of the circle $x^2 + y^2 - 4x + 6y = 9$ is:
(A) $(2, -3)$ (B) $(-2, 3)$ (C) $(-4, 6)$ (D) $(4, -6)$
7. The inverse of $f(x) = 2x - 3$ is:
(A) $\frac{1}{2x-3}$ (B) $\frac{x+3}{2}$ (C) $2x+3$ (D) $-(2x-3)$

Part II: Short Answer Questions

- This part is worth 36 marks out of 50.

1. (7 marks) Find all real solutions of the equation $\sqrt{x + \sqrt{7 - x}} = 1$.
2. (6 marks) Solve the inequality $\frac{4x + 1}{x + 2} \geq 2$ and write your answer in interval form.
3. (7 marks) Ali and Said have been hired to do a specific job. It takes 3 hours and 30 minutes for Said to do the job alone. Working together, they can do in 40% of the time it takes Ali working alone. How long does it take Ali to do the job alone? Write your answer in hours and minutes.
4. (6 marks) Let $f(x) = -2x^2 + 8x + 3$.
 - (a) Write f in standard form.
 - (b) Find the range of f . Write your answer in interval form.
 - (c) What is the maximum value of f ?
5. (4 marks) Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{4 - x}$. Find $(f \circ g)(x)$ and its domain. Write your answer in interval form.
6. (6 marks) Sketch the graph of g

$$g(x) = \begin{cases} -2 - x & \text{if } x < -2 \\ 4 - x^2 & \text{if } -2 \leq x < 0 \\ 2 & \text{if } x \geq 0 \end{cases}$$

and evaluate $g(-3)$, $g(-1)$, $g(0)$, $g(a^2)$ where a is a real number.

M1106 - Test 2 - Fall 2009

Part I: Multiple Choice Questions

- Each Multiple Choice Question is worth 2 marks for a total of 10 marks out of 50.
- Circle the appropriate answer and write it in the table below.

1. Evaluate $\sqrt{-10}\sqrt{-40}$:

- (A) 20 (B) -20 (C) $20i$ (D) $-20i$

2. The remainder in division of $P(x) = 2x^5 - 3x^4 + x^2 + 6$ by $(x + 1)$ is:

- (A) -4 (B) -1 (C) 2 (D) 6

3. The domain of the function $f(x) = \frac{1}{\ln(x^2 + 1)}$ is:

- (A) $(0, \infty)$ (B) $(-\infty, 0) \cup (0, \infty)$ (C) $(-\infty, \infty)$ (D) $(1, \infty)$

4. If $\cos \theta = -\frac{5}{\sqrt{61}}$ and θ is in quadrant II then $\tan \theta$ is:

- (A) $-\frac{6}{5}$ (B) $\frac{6}{5}$ (C) $-\frac{6}{\sqrt{61}}$ (D) $\frac{6}{\sqrt{61}}$

5. The exact value of $\sin\left(\frac{35\pi}{6}\right)$ is:

- (A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Part II: Short Answer Questions

- This part is worth 40 marks out of 50. Simplify your answer when possible.
- To get full marks you have to show all necessary work.
- Write your answer in the space provided after the question.

1. (6 marks) Solve for x : $16^x - 4^{x+1} = 12$.

2. (8 marks) Sketch the graph of the rational function $r(x) = \frac{2x^2 - x - 3}{x^2 + x - 6}$, showing all intercepts and asymptotes.

3. (6 marks) The frog population in pond grows exponentially. The current population is 70 frogs, and the relative growth rate is 24% per year.

(a) Find a function that models the population after t years.

(b) Find the frog population after 5 years.

(c) Find the number of years required for the frog population to reach 300.

4. (7 marks) Solve the inequality $\log_2(4 - x) + \log_2(x + 2) \leq 3$ and write your answer in interval form.

5. (7 marks) Hamdi estimated the angle of elevation to the top of clock tower to be 30° . After walking 40 meter closer to the tower, he found that the angle of elevation to the top of the clock tower was 45° . Find the height of the clock tower.

6. (6 marks) Let $P(x) = x^4 - 3x^3 - x^2 + 13x - 10$.

(a) Factor P into linear and irreducible quadratic factors with real coefficients.

(b) Factor P completely into linear factors with complex coefficients.

M1106 - Final Exam - Fall 2009

1. Simplify: $(x + y)(x^{-1} + y^{-1})^{-1}$
 (A) 1 (B) $\frac{1}{xy}$ (C) $(x + y)^2$ (D) xy (E) None of these
2. The equation of the line which passes through the origin and perpendicular to the line $2x - 4y = 5$ is:
 (A) $y = 2x$ (B) $y = \frac{1}{2}x$ (C) $y = -\frac{1}{2}x$ (D) $y = -2x$ (E) None of these
3. The radius of the circle $x^2 - 2x + y^2 + 8y - 8 = 0$ is:
 (A) 9 (B) 8 (C) 3 (D) 17 (E) None of these
4. If the graph of $f(x) = \sqrt[3]{x}$ is shifted 3 units to the left, reflected in the x -axis, and shifted upward 4 units, the equation of the final graph is:
 (A) $y = 4 - \sqrt[3]{x - 3}$ (B) $y = 4 - \sqrt[3]{x + 3}$ (C) $y = 3 - \sqrt[3]{x - 4}$ (D) $y = 3 - \sqrt[3]{x + 4}$ (E) None of these
5. The imaginary part of $(3 - 2i)(1 + 4i)$ is:
 (A) $10i$ (B) -8 (C) 10 (D) $8i$ (E) None of these
6. The solution of the equation $\log_2(1 + 2^x) = -1$ is:
 (A) $-\frac{1}{2}$ (B) 0 (C) -1 (D) No solution (E) None of these
7. The amplitude of $y = 4 \sin 2x - 3 \cos 2x$ is:
 (A) 1 (B) 5 (C) 7 (D) 25 (E) None of these
8. The domain of the function $f(x) = \log_x 10$ is:
 (A) $(0, \infty)$ (B) $(-\infty, \infty)$ (C) $(0, 1) \cup (1, \infty)$ (D) $(1, \infty)$ (E) None of these
9. The maximum value of $f(x) = -x^2 + 4x - 5$ is:
 (A) 1 (B) -1 (C) 2 (D) -2 (E) None of these
10. If $\cos(\theta) = -\frac{1}{4}$ and $\frac{\pi}{2} < \theta < \pi$, then $\sin(2\theta)$ is:
 (A) $\frac{15}{16}$ (B) $\frac{\sqrt{15}}{8}$ (C) $-\sqrt{15}$ (D) $-\frac{\sqrt{15}}{8}$ (E) None of these
11. Write the expression $\sin(\tan^{-1} x)$ as an algebraic function of x :
 (A) $\frac{x}{\sqrt{x^2 + 1}}$ (B) $\frac{x}{\sqrt{x^2 - 1}}$ (C) x (D) $\sqrt{1 - x^2}$ (E) None of these

Part II: Short Answer Questions

- This part has 12 questions for a total of 78 marks. To get full marks you have to show all necessary work.
- Write your answer in the space provided after the question. Simplify your answer as far as possible.

- (7 marks) A hot cup of coffee cools according to Newton's Law of cooling $T(t) = 35 + 50e^{-0.04t}$, where the temperature T is measured in $^{\circ}C$ and time t in minutes.
 - What is the initial temperature of the coffee?
 - Find the temperature of the coffee after 10 minutes.
 - After how long will the temperature be 65° ?
- (8 marks) Find the amplitude, period and phase shift of the function $g(x) = -2\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)$, then sketch the graph of g in one complete period.
- (5 marks) For $P(x) = x^4 + 2x^3 + 4x^2 - 2x - 5$ list all possible rational zeros and find all real and complex zeros.
- (7 marks) Sketch the graph of $P(x) = x(x+1)^2(x-1)^3$ clearly indicating all y - and x -intercepts and the end behavior.
- (7 marks) Verify the identities:
 - $\frac{\sin 2x}{1 + \cos 2x} = \tan x$
 - $\sin(\sin^{-1} x + \cos^{-1} x) = 1$
- (8 marks) Sketch the graph of the function $r(x) = \frac{x^2 + 3x}{x + 1}$. Find vertical, horizontal and slant asymptotes if any. Show clearly all y - and x -intercepts and the behavior near asymptotes.
- (6 marks) Find exact solution(s) of $\log_2(2 + x) + \log_3 9 = 2\log(2 - x)$
- (5 marks) Solve the inequality $\frac{2}{x - 2} \leq \frac{1}{x}$ and write your answer in interval form.
- (5 marks) A man drove from A to B at a speed 120 km/h. On the way back, he drove at 90 km/h. The total time of the journey was 4 hours and 40 minutes. Find the distance between A and B .
- (7 marks) Let $f(x) = \frac{1}{x^2 - 1}$ and $g(x) = \sqrt{3 - x}$.
 - Find $(f \circ g)$ and its domain in interval form
 - Find g^{-1}
- (7 marks) Find exact solutions of $\cos x = \sin 2x$ for $x \in [0, 2\pi)$.
- (6 marks) A helicopter is flying directly above a straight highway. Two cars, that are 600 m apart, move on the highway on both sides of the helicopter. The angle of depression to the cars is determined to be 40° and 80° . How far is each car from the helicopter?