

SULTAN QABOOS UNIVERSITY - COLLEGE OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

First Interm Examination Fall 2007 07/10/2007

MATH 1106 - Precalculus

General Instructions:

- The duration of this exam is **65 minutes**: 5 minutes to read the instructions and the questions, and 60 minutes to write the answers.
- There are 12 questions in all, of which the first 5 are *multiple choice* questions.
- Answer all questions. Do not give more than one answer to a question.

Marks – Q1: 1 Q2: 1 Q3: 1 Q4: 2 Q5: 2 Q6: 5 Q7: 5 Q8: 5
Q9: 2 Q10: 5 Q11: 5 Q12: 6 **Total Marks: 40**

Instructions for multiple choice questions 1–5: For the multiple choice questions, you are required to write the **answers only**, in the answer booklet. For example, if the answer to a question is part (b) of the given answers, write (b) against that question number. Rough work for the multiple choice questions, if needed, should be done in the last two pages of the answer booklet. Rough work will not be graded.

In questions 1–5, choose one answer for each question that best completes the statement or answers the question.

1. Which of the following pairs of equations describes two perpendicular lines?

- (a) $y - 2x = 2$, $y + 2x = -1$ (b) $y - 2x = 3$, $2y + x = 0$
(c) $y - 2x = 2$, $x - 2y = -1$ (d) $2x + y = -2$, $2y + x = 3$

2. The solution set of the inequality $-4 \leq -4x - 12 < 4$ is

- (a) $(-\infty, -4]$ (b) $[-4, -2)$
(c) $(-\infty, -4) \cup [-2, \infty)$ (d) $(-4, -2]$

3. The solutions of the equation $|1 - 2x| = 12$ are

- (a) $\{-11/2, 13/2\}$ (b) $\{11/2, -11/2\}$
(c) $\{-11/2, -13/2\}$ (d) $\{-13/2, 11/2\}$

4. The solution set of the inequality $\frac{-5}{x+3} > 2$ is

- (a) $(-\infty, -3)$ (b) $(-3, 1/2)$
(c) $(-\infty, -11/2)$ (d) $(-11/2, -3)$

5. The domain of $f(x) = \sqrt{16 - x^2}$ is

- (a) $(-\infty, \infty)$ (b) $[-4, 4]$
(c) $(-\infty, -4] \cup [4, \infty)$ (d) $(0, 4)$

Please turn over for the remaining questions

For questions (6)–(12) below, show all work

6. Simplify: $\left(\frac{x}{x-1} - \frac{3}{x-1}\right)^{-1} \left(\frac{1}{x-1} - \frac{2}{x+1}\right)$

7. Solve: $x - \sqrt{14 - x} = 2$

8. Solve and express the solution in interval form: $\frac{(x-3)(x+2)}{x-1} \leq 0$

9. The solution set of the inequality $x^2 - x + 1 < 0$ is the empty set. Give reason.

10. Find the center and radius of the circle $x^2 + y^2 + 4x - 4y - 1 = 0$.

11. A function f is defined as

$$f(x) = \begin{cases} 1 - x, & \text{if } -1 \leq x < 1 \\ -4, & \text{if } x = 1 \\ x^2, & \text{if } x > 1 \end{cases}$$

Sketch the graph of f , and then find its range.

12. Ahmed and Nabil, working together, can complete a job in 4 hours. Working alone, it would take Ahmed 2 hours longer than Nabil to finish the job. How long does it take Nabil to complete the job?

For questions (6)–(10) below, show all work, and simplify your answers

6. Let $f(x) = \frac{x}{x-2}$, $g(x) = \frac{2x}{x^2+5}$ and $h(x) = \sqrt{5-x}$.
- (a) Find $F(x) = \left(\frac{f}{g}\right)(x)$, and the domain of F in interval form.
- (b) Find $G(x) = (f \circ h)(x)$, and the domain of G in interval form.
7. Sketch the graph of the polynomial function, $P(x) = (x+1)(x-2)^2(3-x)$. Label all intercepts clearly.
8. Let $f(x) = (x+2)^2 - 1$, $x \leq -2$.
- (a) Sketch the graph of f , and show that f is one-to-one.
- (b) Find $f^{-1}(x)$.
9. Let $P(x)$ be a polynomial of degree 4 with rational coefficients. Three of its zeros are: 0 (multiplicity 2) and $2-i$. Find $P(x)$ in which the coefficient of x^3 is 5.
10. Find all intercepts and asymptotes of the rational function $r(x) = \frac{2x - x^2}{x^2 + 2x - 3}$, and sketch its graph.
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SULTAN QABOOS UNIVERSITY - COLLEGE OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

Final Examination Fall 2007 15/12/2007

MATH 1106 - Precalculus

A

Instructions: The duration of this exam is $2\frac{1}{2}$ hours. There are 18 questions in this exam. Answer all questions. Do not give more than one answer to a question.

Marks: Q1–Q6: 2 marks for each question, Q7: 5 + 6, Q8: 6, Q9: 4 + 4, Q10: 4, Q11: 4 + 2, Q12: 1 + 2 + 4, Q13: 8, Q14: 7, Q15: 6, Q16: 6 + 3, Q17: 5 + 4, Q18: 7. Total: 100

Instructions for multiple choice questions 1–6: For the multiple choice questions, you are required to write the **answers only**, in the **answer booklet**. Write one of the choices A, B, C or D, for each question that best completes the statement or answers the question.

- The solution of the equation $\frac{1}{3}(1 - 3x) - \frac{2}{3}(x - 8) = x - 1$, is
(A) $-\frac{3}{2}$ (B) 3 (C) $\frac{9}{4}$ (D) None of these
- The area of a triangle with sides of length 2 and 5, and included angle 30° , is
(A) 5 (B) $\frac{5}{2}$ (C) $\frac{5\sqrt{3}}{2}$ (D) None of these
- Let k be any integer. The solutions of $2 \cos x + 1 = 0$ are
(A) $\frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$ (B) $\frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi$
(C) $\frac{4\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$ (D) None of these
- Which equation does not have a solution?
(A) $\sin(100x) = 0.1$ (B) $\tan x = 20000$
(C) $\cos^2 x - 0.25 = 0$ (D) $\sqrt{5} \sin x - 3 = 0$
- The solutions of $e^{2x} - 3e^x + 2 = 0$ are
(A) 0, $\ln 2$ (B) e, e^2 (C) 1, 2 (D) 0, 2
- Let $f(x) = \log(x - 2)$. Then $f^{-1}(x)$ is
(A) $\frac{1}{\log(x - 2)}$ (B) $e^x + 2$ (C) $10^x + 2$ (D) $\frac{1}{2 + 10^x}$

In Questions 7–18, show all work, and justify your answers

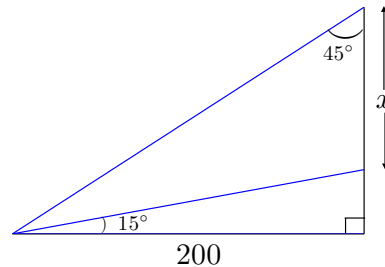
- (a) Solve the equation and check your solution(s): $\log_9(x - 5) + \log_9(x + 3) = 1$
(b) Show that $1/2$ is a zero of the polynomial $P(x) = 2x^3 - 5x^2 + 6x - 2$, and then find all other real or complex zeros of $P(x)$.
- Solve the inequality and write your answer in interval form: $\frac{7 - x}{2 - x} \geq 2$

Please Turn Over for the remaining Questions

9. (a) Find an equation of a straight line passing through $(-2, 1)$ and perpendicular to the line joining the points $(-3, 5)$ and $(1, 2)$.
- (b) For what values of k does the equation $x^2 + y^2 - 2x + 6y + 2k = 0$ represent a circle?
10. An investor has 60000 Rials to invest. An amount of 52000 Rials is invested at 8% and the remaining at $R\%$. If the investor receives a total annual interest of 5000 Rials, find R .
11. (a) Let $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{x+1}{x-2}$. Find $f \circ g$, and its domain in interval form.
- (b) Let $h(x) = \frac{1}{\sqrt{2 \sin x - 1}}$. Find the domain of h in the interval $[0, 2\pi)$.
12. A hot bowl of soup cools according to the Newton's Law of cooling:
 $T(t) = 35 + 50e^{-0.045t}$, where the temperature T is measured in $^{\circ}\text{C}$ and time t in minutes.
- (a) What is the initial temperature of the soup?
- (b) Find the temperature of the soup after 8 minutes.
- (c) After how long will the temperature be 67.5°C ?
13. Sketch the graph of the rational function, $r(x) = \frac{x^2}{(x-1)^2}$.
- Show clearly all intercepts and asymptotes.

14. Without using calculator, show that the exact value of x , in the figure, is

$$200(\sqrt{3} - 1).$$



15. Find the amplitude, period and phase shift of the function

$$y = 3 \cos\left(\frac{\pi}{2}x - \pi\right), \text{ and graph one complete period.}$$

16. Let $\cot \theta = -\frac{4}{3}$, $\sin \theta > 0$.

- (a) Find the exact values of $\sin 2\theta$ and $\cos 2\theta$.
- (b) Using the results in Part (a), find the exact value of $1 + \cos 3\theta$.

17. (a) Prove: $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = 2 \tan 2\theta$

- (b) Rewrite $\sin(2 \tan^{-1} x)$ as an algebraic expression in x .

18. Find the solutions of the following equation in the interval $[0, \pi)$:

$$2 \sin(2x) \cos(3x) = \sin(2x)$$