

SULTAN QABOOS UNIVERSITY - COLLEGE OF SCIENCE

DEPARTMENT OF MATHEMATICS AND STATISTICS

Interm Examination 1 Fall 2006 09/10/2006

Math 1106 - Precalculus

Instructions: The duration of this exam is 1 hour — The total marks for this exam is 40 — *Do all problems*
— To get full marks you have to show all necessary work.

Marks— Q1: 4, Q2: 6, Q3: 6, Q4: 4 + 4, Q5: 6, Q6: 5, Q7: 5

1. Simplify and eliminate any negative exponents: $\frac{(2x^2y)^{-1}}{(-3x^{-2}y^3)^3}$
 2. Simplify: $\frac{t-2}{2t^2-5t+3} - \frac{3}{t^2-1}$
 3. Solve the equation: $x + \sqrt{2x+5} = 5$
 4. Solve the following inequalities and express the solutions in interval form:
 - (a) $\frac{1}{4}(2x-1) - x < \frac{x}{6} - \frac{1}{3}$
 - (b) $|2-3y| \geq 7$
 5. Find the domain of $f(x) = \frac{3}{x} + \sqrt{\frac{1-x}{x+2}}$, and write your answer in interval form.
 6. Starting from the graph of $f(x) = \sqrt{x}$, use suitable transformations to sketch the graph of
$$g(x) = 3 - \sqrt{x-4}$$

Find the range of g from your graph, and write it in terms of interval.
 7. An investor invests 38000 rials at 6% annual interest rate and x rials at a rate of $8\frac{1}{2}\%$. If he receives a total annual interest of 5000 rials, find x .
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Marks— Q1: 8, Q2: 5, Q3: 5, Q4: 9, Q5: 4, Q6: 4, Q7: 5

- Let $f(x) = \frac{x}{x+1}$ and $g(x) = \sqrt{2x-1}$.
 - Find $f^{-1}(x)$
 - Find (f/g) , $(f \circ f)$, and their domains.
- Find a polynomial of degree 3 with integer coefficients that has zeros 1 and $2+i$, and leading coefficient 2.
- Let $P(x) = 3x^3 - 8x^2 + 3x + 2$
 - Show that $x - 2$ is a factor of $P(x)$.
 - Factor $P(x)$ completely and find all zeros.
- Given the function $h(x) = \frac{x^2 - 2x + 1}{x - 2}$,
 - Find the x -intercept(s) and y -intercept, if any.
 - Find all asymptotes, if any.
 - Sketch the graph of $y = h(x)$.
- Solve for u : $\log_2 u + \log_2(u + 2) = 3$
- Simplify *without using a calculator*: $\frac{\log_3 x + \log_9 x}{\log_{27} x - \log_{81} x}$
- A culture of bacteria contains 2000 bacteria initially. After 20 minutes, the bacteria count is 6000.
 - Find a function of the form $n(t) = n_0 e^{rt}$ that models the number of bacteria after t minutes.
 - After how many minutes will the culture contain 4200 bacteria?

MATH 1106 - Precalculus

Instructions:

- The duration of this exam is $2\frac{1}{2}$ hours.
- There are 14 questions in this exam. *Answer all questions.*
- To get full marks you have to show all necessary work.

Marks - Q1: 5 + 5, Q2: 2 + 6, Q3: 4 + 4, Q4: 10, Q5: 5, Q6: 4, Q7: 4 + 4, Q8: 3 + 3, Q9: 5, Q10: 6, Q11: 5 + 6, Q12: 6, Q13: 5, Q14: 2 + 6. Total Marks: 100

1. Solve the following equations:

(a) $\frac{3x}{x + \frac{1}{2}} = \frac{2x - 1}{x + 2}$

(b) $u^{1/2} - 8u^{1/4} - 9 = 0$

2. Solve the following inequalities. Write the solution in **interval** notation.

(a) $|3 - x| + 2 \geq 0$

(b) $\frac{x + 1}{2x - 3} \leq \frac{1}{3}$

3. Let $g(x) = 2[(x - 1)^2 - 1]$.

(a) Sketch the graph of g , and write its range.

(b) Find the inverse of g , for $x \leq 1$.

4. Find all intercepts and asymptotes, if any, and then sketch the graph of the function

$$h(x) = \frac{x^2 + x - 2}{x^2 + 3x}$$

5. Solve for z , and express the solutions in the form $a + ib$:

$$(1 + i)z^2 - 2iz + 1 - i = 0$$

6. Find the domain of $f(u) = \frac{\ln u}{1 + \ln u}$.

7. A sum of 5000 Rials is invested at an interest rate of 9% per year, compounded quarterly.

(a) Find the amount after 5 years.

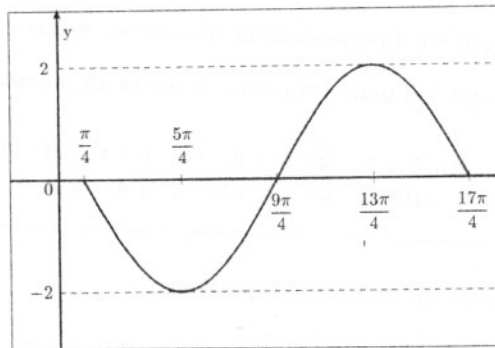
(b) Find the time required for the investment to grow to 8000 Rials.

8. The graph of one complete period of a sine curve is shown in the figure.

(a) Find the amplitude, period and phase shift.

(b) Write an equation that represents the curve, in the form

$$y = a \sin k(x - b)$$



9. From the top of a light house 200 meters tall, the angles of depression of two ships approaching it from the **same** direction are 30° and 45° . Find the distance between the ships.

10. Solve the triangle ABC for which $a = 6$, $b = 4$, and $\angle A = 110^\circ$.

11. (a) Write $\sin(\tan^{-1} x - \sin^{-1} x)$ as an algebraic expression in x .

(b) Prove: $\tan^2\left(\frac{x}{2} + \frac{\pi}{4}\right) = \frac{1 + \sin x}{1 - \sin x}$

12. Solve for x in the interval $[0, 2\pi)$: $\sin x - \cos x + 1 = 0$

13. Express the complex number $-4\sqrt{3} + 4i$ in trigonometric form.

14. Let $z = 8\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$.

(a) Find z^3 , and write it in the form $a + ib$.

(b) Find the cube roots of z in **trigonometric form**, and graph the roots in the complex plane.
