Date: 11 October 2005 Total Marks: 40 Time: 1 hour.

Guidelines:
1. Read all questions first.
2. Answer all 6 questions and justify your answer in order to get full marks.
3. Do your best in this test, and keep using the Pre-Calculus Drop-in Center faciliti
   and resources in order to improve your performance for the next exams.

1. [5 Marks] Simplify the expression $x - \dfrac{y}{xy^{-1} + yx^{-1}}$.

2.

(a)[4.5 Marks] Find the domain of $f(x) = \sqrt{\dfrac{2 - |x|}{x^2 + 1}}$.

(b)[1.5 Marks] Test the equation $y - \sqrt{\dfrac{2 - |x|}{x^2 + 1}} = 0$ for all three types of symmetry

3.
(a)[6 Marks] Solve the equation $x + \sqrt{4x - 8} = 5$.

(b)[6 Marks] Solve the inequality $\dfrac{x + 1}{x - 1} \geq \dfrac{x - 1}{x + 1}$ and express your answer in interv
form.

4.[6 Marks] If the circle is defined by the equation $x^2 + y^2 - 2x + \mathbf{c}y - 15 = 0$ ha
radius 5, find all values of the constant **c**.

5. Let $\mathbf{g}(x) = 3 - |x - 2|$.
(a)[4 Marks] Starting from the graph of $y = |x|$, use appropriate transformations t
sketch the graph of **g**. At each step show all intercepts on your graph.

(b)[2 Marks] From the graph, find the range of **g**.

6.[5 Marks] A father is four times as old as his daughter. Two years ago, he was fiv
times as old as she. How old is the father now?

Date: 22 November 2005   Total Marks: 40   Time: 1 hour.

Guidelines:
1. *Read all questions first.*
2. *Answer all 5 questions and justify your answer in order to get full marks.*
3. *Do your best in this test, and keep using the Pre-Calculus Drop-in Center facilities and resources in order to improve your performance for the Final Exam.*

1. Solve for x:

(a) [5 Marks] $\ln(x^2 + 4) = 3\ln 2 + \ln(x + 3)$.

(b) [5 Marks] $3^{2x+1} - 3^{x+2} - 12 = 0$.

2. Let $f(x) = \dfrac{2x}{3x - 1}$.

(a) [1 Mark] Find the domain of f.

(b) [5 Marks] Find $(f \circ f)(x)$ and its domain.

(c) [4 Marks] Find the inverse of the function $h(x) = \dfrac{3x^3 + 4}{4x^3 - 3}$.

3.
(a) [6 Marks] Find all real and complex zeros of $q(x) = 2x^3 + 5x^2 + 10x + 4$.

(b) [4 Marks] Find a polynomial $P(x)$ of degree 3 with integer coefficients so that $P(-2) = 0$, $P(4i) = 0$, and $P(2) = 80$.

4. [5 Marks] Find all intercepts (if any) and asymptotes (if any) of $r(x) = \dfrac{4x^2 + 1}{x^2 + x - 6}$.
(**Do not sketch the graph**).

5. [5 Marks] Among all rectangles that have perimeter 40 ft, find the width **w** and the length **L** of the rectangle whose area is maximum.

Sultan Qaboos University — College of Science
Department of Mathematics and Statistics
MATH 1106 — Precalculus — Final Exam Fall 2005

Date: 21 December 2005   Total marks: 90   Time: 150

Guidelines:
1. *Read all questions first.*
2. *Answer all questions and justify your answers in order to get full marks.*

1. Consider the equation $\cos(2x) - \sin(x) = 0$.
   (a) [6.5 marks] Find the solutions in the interval $[0, 2\pi)$.
   (b) [1.5 marks] Find all the solutions.

2. Consider the complex number $z = \sqrt{3} + i$.
   (a) [3 marks] Write z in trigonometric form $r(\cos\theta + i\sin\theta)$.
   (b) [5 marks] Find the cube roots of z.

3. [5 marks] Sketch in the complex plane the set $\{z = a + bi : |z| < 2, \ a \geq b\}$.

4. [6 marks] Rewrite $\sin(\cos^{-1} x - \tan^{-1} x)$ as an algebraic expression in x.

5. [6 marks] Simplify the expression $\dfrac{12(\log_4 |x|)(\log_8 |x|)}{\log_2 (x^2)}$.

6. [6 marks] Consider the function $f(x) = \ln(1 - x) - \sqrt{2 - |x|} + \dfrac{x}{2x + 1}$.
   State the domain of f in interval form.

7. Consider the rational function $r(x) = \dfrac{2x(x^2 + 1)}{(x + 1)(x - 1)}$.
   (a) [7 marks] Find all intercepts, all asymptotes, the end behavior, and the behavior near vertical asymptotes of r.
   (b) [4.5 marks] Sketch the graph of r and indicate clearly all intercepts and asymptotes on your graph.

8. Consider the function $g(x) = 2\cos(\frac{1}{2}x - \pi)$.
   (a) [5 marks] Find the amplitude, period, phase shift, and the range of g.
   (b) [4 marks] Graph one complete period of g and indicate all x-intercepts on your graph.

9. [6 marks] Write the expression $-\sqrt{3}\sin(2x) + \cos(2x)$ in the form $k\sin(2x + \phi)$, where k a positive constant.

10. Consider the circle $x^2 + y^2 - 4x = 0$.
   (a) [5 marks] Find the equation of the line passing through the center of the circle and perpendicular to the line $y - x = 0$.
   (b) [6 marks] Find the points of intersection of the line $y = 2 - x$ with the circle.

11. Consider a triangle ABC with sides $a = 8$ and $b = 2$, and included angle $\angle C = 120°$.
   (a) [3 marks] Find the area of the triangle.
   (b) [4.5 marks] Solve the triangle.

12. A cup of coffee is served hot at a dinner. It starts to cool according to Newton's Law of Cooling so that its temperature at time t is given by: $T(t) = 70 + 130e^{-0.05t}$, where T is measured in $°F$ and t in minutes.
   (a) [3 marks] Determine the temperature of the surroundings, and the initial temperature the coffee.
   (b) [3 marks] How long after serving will the temperature of the coffee be $100°F$.