Phys 2107 Physics for Engineers I

Test I *Date*: 01/10/2007 *Time*:16:00-17:00

ID:	Name:	Sec:	Score:

Please check that you have 3 questions. Take $g = 10 m/s^2$. (Total score = 50)

1. Let $\vec{r} = x(t)\hat{i} + y(t)\hat{j}$ represents the position vector of a particle where $x(t) = -2t + 3t^2$ and $y(t) = 4t - t^2$ are measured in meters and t in second.
   (a) Calculate the average velocity between the times $t = 1s$ and $t = 2s$.
   (b) Find the velocity $\vec{v}$ when the particle reaches its maximum y coordinate.
   (c) Find the magnitude and the direction of the position vector when $y = 3m$.
   (d) Find the acceleration in unit vector notation at $t = 0$. **(18 points)**

(4) a) $\vec{V}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(2) - \vec{r}(1)}{2-1} = (8\hat{i} + 4\hat{j}) - (\hat{i} + 3\hat{j}) = 7\hat{i} + \hat{j}$ m/s

(4) b) $V_y = \frac{dy}{dt} = 0 \Rightarrow 4 - 2t = 0, \; t = 2s$

$\vec{V} = \frac{d\vec{r}}{dt} = 10\hat{i}$

(6) c) $y = 3 = 4t - t^2 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow t = 1s, \; t = 3s$

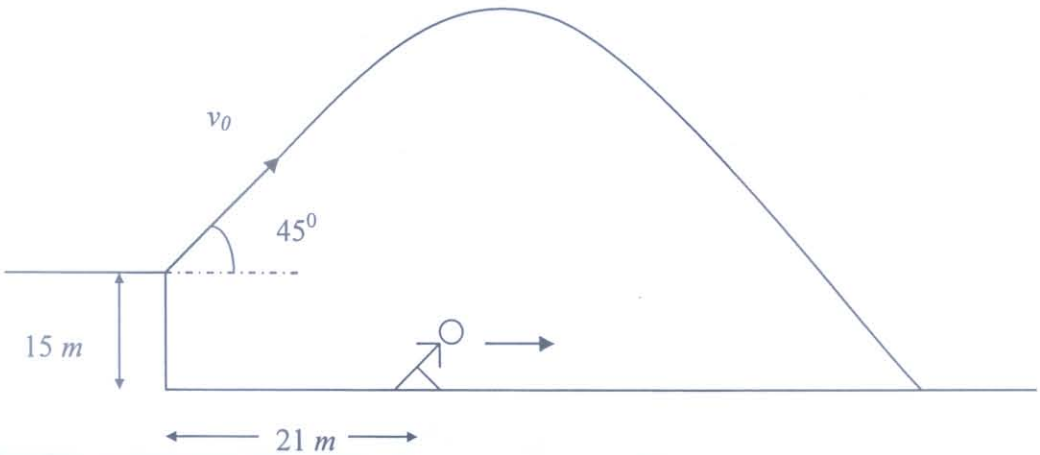
$\vec{r}(1) = \hat{i} + 3\hat{j}, \; |\vec{r}| = \sqrt{10} \approx 3.2m, \; \theta = \tan^{-1} 3, \; \theta \approx 72^\circ$

$\vec{r}(3) = 2\hat{i} + 3\hat{j}, \; |\vec{r}| = \sqrt{9 + 441} \approx 21m, \; \theta = \tan^{-1} \frac{3}{21} = 8.1^\circ$

(4) d) $\vec{a} = \frac{d\vec{V}}{dt} = 6\hat{i} - 2\hat{j}$ const vector, independent of time.

2. A ball is thrown upward from a $15m$ tall building at an angle 45^0 above the horizontal. At the same instant a player on the ground $21m$ away from the building starts running from rest to catch the ball. The player runs with a constant acceleration and catches the ball after 3 second just before the ball hits the ground.

(a) Calculate the initial speed of the ball.
(b) Find the maximum height of the ball from the ground.
(c) Compute the magnitude and direction of the velocity at $t = 2s$.
(d) Find the acceleration of the player. **(18 points)**



$x = v_{ox} t = v_0 \cos 45^\circ t$

$y = 15 + v_{oy} t - 5t^2 = 15 + v_0 \sin 45^\circ - 5t^2$

$v_x = v_{ox}$, $v_y = v_{oy} - gt$

(5) (a) at $t = 3s$, $y = 0 \Rightarrow 0 = 15 + 3v_{oy} - 45 \Rightarrow v_{oy} = 10 m/s$

$v_0 = \frac{10}{\sin 45^\circ} \simeq 14 m/s$

(5) (b) $y_{max} = 15 + v_{oy} t_{max} - 5 t_{max}^2$, $v_y = 0 \Rightarrow t = \frac{v_{oy}}{g} = 1s$

$y_{max} = 15 + 10 - 5 = \underline{20 m}$

(4) (c) $v_x = v_{ox} = \frac{v_0}{\cos 45^\circ} = 10 m/s$, $v_y = 10 - 20 = -10 m/s$

$v_0 = 10\sqrt{2} = \underline{14 m/s}$

(4) (d) $x_{ball} = 3 \times 10 = 30 m$ $\quad x_{ball} = x_{player} \Rightarrow \boxed{a = 2 m/s^2}$

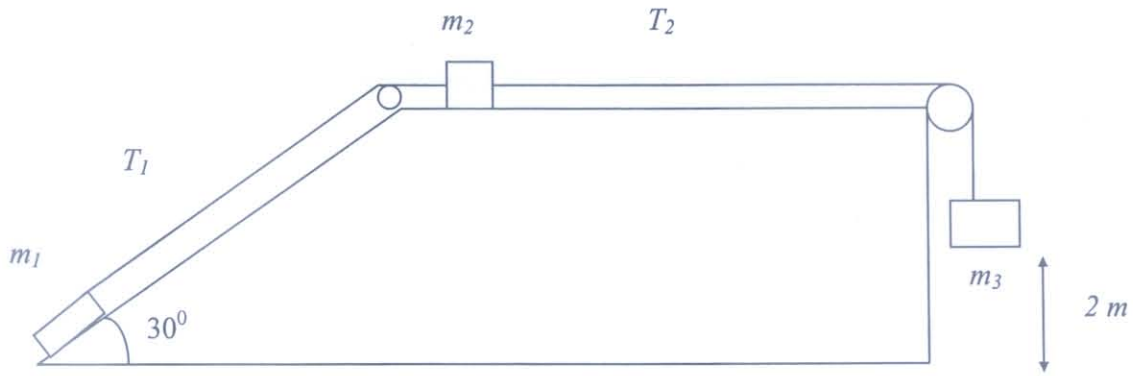
$x_{player} = 21 + \frac{1}{2} a (3)^2$

3. Three blocks of masses $m_1 = 2kg$, $m_2 = 3kg$ and $m_3 = 5kg$ are connected to each other by cords over pulleys as shown in the figure. The system is released from rest when $5kg$ block is $2m$ above the ground. Surfaces are frictionless.

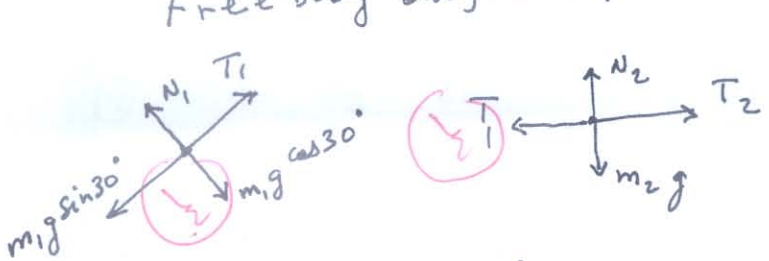
(a) Calculate the magnitude of the acceleration of the system.

(b) Find the tensions T_1 and T_2 in the interconnecting cords.

(c) Find the speeds of the blocks just before the $5kg$ block hits the ground. **(14 points)**



Free body diagrams:



(a) $T_1 - m_1 g \sin 30^\circ = m_1 a$

$T_2 - T_1 = m_2 a$

$m_3 g - T_2 = m_3 a$

$$a = \frac{m_3 g - m_1 g \sin 30^\circ}{m_1 + m_2 + m_3} = \frac{(5-1)10}{10} = 4\ m/s^2$$

(b) $T_1 = 2(5+4) = 18\ N$

$T_2 = 18 + 3 \times 4 = 30\ N$

(c) $v^2 = 2ax \Rightarrow v = \sqrt{2 \times 4 \times 2} = 4\ m/s$ for all blocks