

- ▶ The duration of this exam is 60 minutes and is worth 40 marks.
- ▶ Do all problems. To get full credit: show your work, and mention theorems when appropriate.

1. [5+5 marks] Evaluate the following integrals:

(a) $\int_{1/4}^0 \tan^3 \pi x \sec^4 \pi x \, dx$

(b) $\int_1^0 x^2 \sqrt{1-x^2} \, dx$

2. [4+4 marks]

(a) Determine whether the sequence $\left\{ (-1)^n \frac{n}{\ln \sqrt{n}} \right\}_{n=1}^{\infty}$ converges or diverges.

(b) Let $a_n = (100)^n \frac{2^n 3^n n!}{2^n}$. Find out whether the sequence a_n is eventually increasing or decreasing.

3. [4+4 marks] Determine whether the following series converge or diverge. For convergent series, find their sum.

(a) $\sum_{k=1}^{\infty} \frac{5(-1)^k 3^k}{(2)_{2k}}$

(b) $\sum_{k=0}^{\infty} (e^{-k} - e^{-(k+1)})$

4. [6 marks] Use the Integral Test to find out whether $\sum_{k=1}^{\infty} \frac{1}{1+k^2}$ converges or diverges.

5. [8 marks] Evaluate $\int_{-\infty}^1 \frac{1}{3x^2+1} \, dx$