Sultan Qaboos University - College of Science
Department of Mathematics and Statistics
MATH 2108, Calculus II
Test2–SPRING 2008

Date: 14 April 2008 | Total marks: 40 | Time: 1 hour

SHOW ALL YOUR WORK.

1. [3+3=6 Marks] Find the limits (if exist) of the following sequences $\{a_n\}$:
   (i) $a_n = 1 + (-1)^n$ (ii) $a_n = \dfrac{2\ln n + 1}{1 - 3\sqrt{n}}$
   Justify your answers.

2. [4 Marks] Find all values of x for which the series $\sum_{k=1}^{\infty} 2^{5-k} \sin^{k+1} x$ converges, and find the sum of the series for those values of x.

3. [4+4+4=12 Marks] Determine whether the following series converge or diverge:
   (a) $\sum_{k=1}^{\infty} \dfrac{1}{k^2 + 3k + 2}$
   (b) $\sum_{k=1}^{\infty} \dfrac{(2k)!}{k!k!3^k}$
   (c) $\sum_{k=1}^{\infty} \dfrac{(-1)^k}{\sqrt{k} + \sqrt{k+1}}$

4. [4 Marks] Show that $\sum_{k=1}^{\infty} \dfrac{1}{(k+3)(k+4)} = \dfrac{1}{4}$.

5. [6 Marks] Evaluate the integral $\int \dfrac{dx}{x^2\sqrt{x^2+16}}$.

6. [8 Marks] Express $\dfrac{4x^2 + x - 2}{x^3 - x^2}$ as a partial fraction. Hence evaluate the improper integral $\int_{\frac{1}{2}}^{1} \dfrac{4x^2 + x - 2}{x^3 - x^2}\, dx$.

END OF PAPER