

SULTAN QABOOS UNIVERSITY – COLLEGE OF SCIENCE  
DEPARTMENT OF MATHEMATICS AND STATISTICS  
TEST 2 of MATH 2108 - APRIL 24<sup>th</sup> (SPRING 2007)

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**Instructions:**

- ▶ The duration of this exam is 60 minutes and is worth 40 marks.
  - ▶ *Do all problems.* To get full credit: show your work, and mention theorems when appropriate.
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1. 4 marks Determine whether the sequence  $\left\{ \frac{2^n}{n!} \right\}_{n=1}^{\infty}$  converges or diverges.

2. 4 marks Show that the series  $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)}$  converges and find its sum.

3. 3+4 marks

(a) Show that  $\sum_{k=0}^{\infty} x^k$  converges if  $|x| < 1$  and diverges when  $|x| \geq 1$ .

(b) Evaluate  $\sum_{k=0}^{\infty} \left( \frac{1}{2^k} + \frac{(-1)^k}{5^k} \right)$ .

4. 5 marks For what values of  $p$  does the improper integral  $\int_1^{\infty} \frac{1}{x^p} dx$  converge or diverge. Show your work and explain clearly your statements. Give all details.

5. 5 marks Find the  $n$ th MacLaurin polynomial for  $f(x) = \ln(1+x)$ . Use your result with  $n = 4$  to approximate  $\ln 2$ .

6. 15 marks Determine whether the following series converge or diverge.

(a)  $\sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^k$

(b)  $\sum_{k=1}^{\infty} \frac{k!3^k}{(2k+1)!}$

(c)  $\sum_{k=1}^{\infty} \frac{5\sqrt{k} + 100}{2k^2\sqrt{k} + 9\sqrt{k}}$

(d)  $\sum_{k=1}^{\infty} (-1)^k \frac{1}{k \ln(k+1)}$  (indicate if the series is absolutely or conditionally convergent).