MODEL SOLUTIONS
M2107 – TEST2 – FALL10

① Implicit differentiation gives

$$-\frac{1}{\sqrt{1-x^2y^2}}\left(y+xy'\right)+2x=1+y'$$

$\therefore$ The slope $m=y'(0,\frac{\pi}{2})$ will be given by

$$(-1)\left(\frac{\pi}{2}+0\right)+0=1+m$$

$$\Rightarrow \quad m=-\left(1+\frac{\pi}{2}\right)$$

Equation of the tangent line is

$$y-\frac{\pi}{2}=-\left(1+\frac{\pi}{2}\right)x$$

$$\left[\text{or } y=-\left(1+\frac{\pi}{2}\right)x+\frac{\pi}{2}\right]$$

② Let $f(x)=x^{\frac{1}{4}}$

$$\therefore \quad f'(x)=\frac{1}{4x^{3/4}}$$

$$x_0=16$$

$$\therefore \sqrt[4]{15.4}\approx f(16)+f'(16)(15.4-16)$$

$$=2+\frac{1}{32}(-0.6)$$

$$=1.98125$$

$\longrightarrow$

③ (a) The given function is continuous in $[0, \sqrt{\pi}]$ and differentiable in $(0, \sqrt{\pi})$.

Also, $f(0) = \sin(0) = 0$; $f(\sqrt{\pi}) = \sin \pi = 0$

$\therefore f(0) = f(\sqrt{\pi})$

Thus, hypotheses of Rolle's Theorem are satisfied.

(b) It follows from the Rolle's Theorem:

$$f'(c) = 0, \quad c \in (0, \sqrt{\pi})$$

or $$2c\cos(c^2) = 0$$

$\Rightarrow$ Either $c = 0 \notin (0, \sqrt{\pi})$

or

$$\cos(c^2) = 0 \Rightarrow c^2 = \frac{\pi}{2}, \frac{3\pi}{2} + 2k\pi$$

(k an integer)

$\therefore c = \sqrt{\frac{\pi}{2}} \in (0, \sqrt{\pi})$ is the desired value.

④ $$\lim_{x \to 0} \frac{\ln(1+x^2)}{e^{2x} - 2e^x + 1} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 0} \frac{\left(\frac{2x}{1+x^2}\right)}{2e^{2x} - 2e^x} \quad \left(\frac{0}{0}\right)$$ (by L'Hôpital rule)

$$= \lim_{x \to 0} \frac{2(1+x^2) - 4x^2}{(4e^{2x} - 2e^x)(1+x^2)^2}$$ (again by L'Hôpital rule)

$$= \frac{2-0}{(4-2)(1)^2} = 1$$

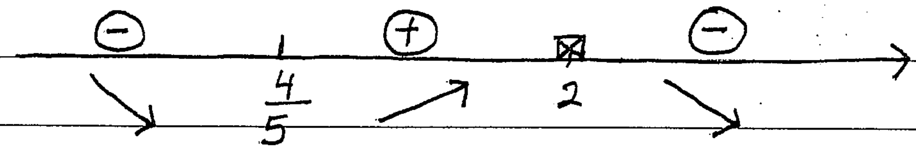
$\longrightarrow$

5 (a) Critical numbers of h are given by:

$5x - 4 = 0 \Rightarrow \boxed{x = \frac{4}{5}}$ and $2 - x = 0 \Rightarrow \boxed{x = 2}$

(b) Let us find signs of $h'(x)$ on the real line:

Sign of $h'(x)$



$\therefore$ h is increasing in $\left(\frac{4}{5}, 2\right)$

h is decreasing in $\left(-\infty, \frac{4}{5}\right) \cup (2, \infty)$

h has a local Minimum at $x = \frac{4}{5}$

h has a local maximum at $x = 2$

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6 (a) $f'(x) = 2x + \frac{K}{x}$, $f''(x) = 2 - \frac{K}{x^2}$

(b) For $K < 0$, clearly $f''(x) > 0$ for all $x > 0$

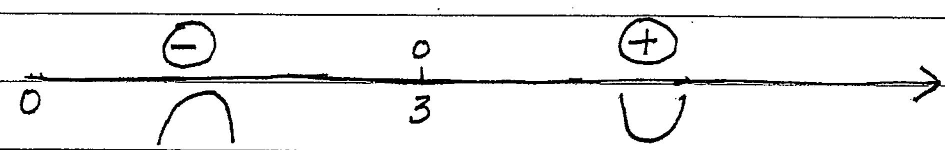
Therefore, there is no change of concavity.

(c) When $K = 18$, we have

$$f''(x) = \frac{2x^2 - 18}{x^2} = \frac{2(x-3)(x+3)}{x^2}$$

$f''(x) = 0 \Rightarrow x = 3$ ($\because x = -3 \notin$ Domain of f)

Sign of $f''(x)$



Since the concavity changes as we pass through $x = 3$, there is an inflection point at $x = 3$.

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KEYS TO MCQs

Q-NO	VERSION I	VERSION II
$\boxed{7}$	(C)	(D)
$\boxed{8}$	(B)	(A)
$\boxed{9}$	(D)	(C)
$\boxed{10}$	(C)	(D)
$\boxed{11}$	(B)	(B)