# SULTAN QABOOS UNIVERSITY
# DEPARTMENT OF MATHEMATICS AND STATISTICS
**22 May 2010**

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## MATH 2107 CALCULUS I
### Spring 2010 Final Examination (VERSION I)
### (Time allowed: 150 minutes)

**NAME:**________________________________**ID#:**________________ **Section:**____

**Instructions:**

- This exam has 15 pages including the front page. There are 21 questions (pages 2–15).
- The empty page (Page 16), at the end, is for rough work and will not be marked.
- Write your name, ID number and Section number on this page. Write your ID number at the top of each sheet.
- Attempt all questions, writing your answer in the space below the statement of the question. For questions 1–11 show all your work.
- Do not give more than one answer to a question.
- For **Multiple Choice Questions, Circle the correct answer**.
- Please DO NOT SEPARATE the pages of this booklet.

**DO NOT WRITE IN THIS BOX!**

| **Question** | Max Marks | Score |
|---|---|---|
| **1** | 6 | |
| **2** | 5 | |
| **3** | 5 | |
| **4** | 6 | |
| **5** | 5 | |
| **6** | 3 | |
| **7** | 9 | |
| **8** | 15 | |
| **9** | 7 | |
| **10** | 6 | |
| **11** | 13 | |
| **12–21** | 20 | |
| **TOTAL** | 100 | |

**1.** $\boxed{\text{6 marks}}$ Find the value of the constant $b$ that makes the following function continuous at $x = 0$ :

$$h(x) = \begin{cases} 2b - x\,, & x \le 0 \\ \dfrac{e - e^{\cos x}}{x^2 - 2x^3}\,, & x > 0 \end{cases}$$

**2.** $\boxed{\text{5 marks}}$ Suppose that $\lim_{x \to 3} f(x) = 3$, $\lim_{x \to 3} g(x) = -2$, and $g$ is continuous at $x = 3$. Find the following:

(i) $g(3)$

(ii) $\lim_{x \to 3} \left( \dfrac{x-3}{\sqrt{x}-\sqrt{3}} \right) g(x)$

(iii) $\lim_{x \to 3} g(f(x))$

**3.** $\boxed{\text{5 marks}}$ Find an equation of the tangent line to the curve $y - x^2 - \sin^{-1}(xy) = 2e$ at $x = 0$.

**4.** $\boxed{3 + 3 \text{ marks}}$ Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ :

(a) $y = e^2 + \sqrt{2x + \sinh(3x^2)}$

(b) $y = \ln\left[\left(\dfrac{x^7}{x^4 + 1}\right)^{1/3}\right]$ [**Note:** Use properties of logarithms]

**5.** $\boxed{\text{5 marks}}$ Use the Mean Value Theorem to show that $|a| \le |\tan a|$ for $-\dfrac{\pi}{2} < a < \dfrac{\pi}{2}$.

**6.** $\boxed{\text{3 marks}}$ Use the Intermediate Value Theorem to show that the equation $4 - x = 2^x$ has at least one solution in $[1,\ 2]$.

**7.** $\boxed{\text{9 marks}}$ An open box is to be made from a 8 feet by 3 feet piece of metal sheet by cutting out squares of equal size from the four corners and folding up the sides. Find the largest volume that the box can have.

**8.** $\boxed{\text{15 marks}}$ Let $f(x) = 3x^5 - 5x^3$ .

(a) Find the critical numbers, intervals of increase and decrease, and the points of local extrema of $f$ .

(b) Find the intervals of concavity and the $x-$values of inflection points of $f$ , if any.

(c) Find the $x$ and $y$ intercepts of $f$, if any.

(d) Sketch the graph of $f$ , in the next page (Page 9)


y
6
4
2
x
−4
−3
−2
−1
1
2
3
−2
−4
−6

**9.** $\boxed{\text{7 marks}}$ Use Riemann sums and a limit to compute the exact area under the curve $y = 2x^2 + 3x$ on $[1,\ 3]$.

**10.** (a) $\boxed{\text{3 marks}}$ Using the definitions of hyperbolic functions, show that

$$1 + 2\sinh^2(2x) = \cosh(4x)$$

(b) $\boxed{\text{3 marks}}$ Given that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$, find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sinh^{-1} x\right)$.

**11.** $\boxed{3 + 4 + 6 \text{ marks}}$ Evaluate:

(a) $\displaystyle\int_1^4 \frac{2-x}{x^{3/2}}\,\mathrm{d}x$

(b) $\displaystyle\int \left(1 - x^2 + e^{-2x}\right)^{13} \left(x + e^{-2x}\right)\,\mathrm{d}x$

(c) $\displaystyle\int_0^{\pi} \frac{\sin x}{3 + |\cos x|}\,\mathrm{d}x$

The remainder of this exam consists of **Multiple Choice** questions. Circle the correct answer for each question. **No partial credit will be given.** (2 marks for each question)

**12.** The horizontal asymptotes of $f(x) = \dfrac{x}{\sqrt[3]{x^3+8}}$ are

**(A)** $y = 1\,,\ y = -1$ **(B)** $y = \dfrac{1}{2}$ **(C)** $y = 1$ **(D)** none of them

**13.** $\displaystyle\lim_{x\to 1^+} \frac{\sin(1-x)}{1-x}$ is

**(A)** 1 **(B)** $-1$ **(C)** $-\infty$ **(D)** $\infty$

**14.** If $f(x) = 2x^5 + x^3 + x + 3$ has the inverse $g(x)$, then $g'(3)$ is equal to

**(A)** 1 **(B)** 0 **(C)** $\dfrac{1}{10x^4 + 3x^2 + 1}$ **(D)** does not exist

**15.** The area of a circular region is increasing at the rate of $96\pi$ m$^2$ per second. When the area of the region is $64\pi$ m$^2$, how fast, in metres per second, is the radius of the region increasing?

**(A)** 6 **(B)** 8 **(C)** 16 **(D)** $4\sqrt{3}$

**16.** $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\int_x^0 \cos(2\pi t)\,\mathrm{d}t$ is

**(A)** 0 **(B)** $-\dfrac{1}{2\pi}\sin x$ **(C)** $\dfrac{1}{2\pi}\cos(2\pi x)$ **(D)** $-\cos(2\pi x)$

**17.** If $g(x) = \begin{cases} x\,, & x \le 1 \\ \dfrac{1}{x}\,, & x > 1 \end{cases}$ , then $\displaystyle\int_0^e g(x)\,\mathrm{d}x$ is

**(A)** $0$ **(B)** $\dfrac{3}{2}$ **(C)** $e$ **(D)** $\dfrac{1}{2} + e$

**18.** The vertical asymptote(s) of $f(x) = \dfrac{x-1}{x^2+2x-3}$ are

**(A)** $x = -1$ and $x = 3$ **(B)** $x = 1$ and $x = -3$ **(C)** $x = -3$ **(D)** none of them

**19.** Let $f(x) = \sqrt{4 + \sin x}$ . The approximate value of $f$ at $x = 0.12$ using the linear approximation with $x_0 = 0$ , is

**(A)** $2$ **(B)** $2.06$ **(C)** $2.03$ **(D)** $2.12$

**20.** The exact value of $\coth(\ln 3)$ is

**(A)** $0$ **(B)** $\dfrac{4}{5}$ **(C)** $\dfrac{5}{4}$ **(D)** none of them

**21.** The expression $\dfrac{1}{50}\left(\sqrt{\dfrac{1}{50}} + \sqrt{\dfrac{2}{50}} + \sqrt{\dfrac{3}{50}} + \cdots + \sqrt{\dfrac{50}{50}}\right)$ is a Riemann sum approximation, with $n = 50$ , for

**(A)** $\displaystyle\int_0^1 \sqrt{\frac{x}{50}}\,\mathrm{d}x$ **(B)** $\displaystyle\int_0^1 \sqrt{x}\,\mathrm{d}x$ **(C)** $\displaystyle\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,\mathrm{d}x$ **(D)** $\displaystyle\frac{1}{50}\int_0^{50} \sqrt{x}\,\mathrm{d}x$

THE END OF THE EXAM

**This page is for rough work. It will not be graded.**