SULTAN QABOOS UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS
09 January 2010

MATH 2107 CALCULUS I

Fall 2009 Final Examination (VERSION I)

(Time allowed: 60 minutes)

NAME:________________________________**ID#:**________________ **Section:**____

Instructions:

- This exam contains 13 pages and 18 questions. **The empty pages at the end are for rough work and will not be marked**.
- Write your name, ID number and Section number on this page. Write your ID number at the top of each sheet.
- Attempt all questions, writing your answer in the space below the statement of the question. For questions 1–8 show all your work.
- Do not give more than one answer to a question.
- For **Multiple Choice Questions, Circle the correct answer**.
- Please DO NOT SEPARATE the pages of this booklet.

DO NOT WRITE IN THIS BOX!

Question	Max Marks	Score
1	9	
2	10	
3	11	
4	10	
5	15	
6	7	
7	9	
8	9	
9–18	20	
TOTAL	100	

1. (a) $\boxed{\text{3 marks}}$ Find $\displaystyle\lim_{x\to 0^-} \frac{3\sin x}{1-\cos(2x)}$

(b) $\boxed{\text{6 marks}}$ Let $f(x) = \begin{cases} \dfrac{\sin(2x)}{x}, & x < 0 \\ a\,, & x = 0 \\ b^2 e^x + b\,, & x > 0\,. \end{cases}$

Find values of the constants a and b for which f is continuous at $x = 0$.

2. (a) $\boxed{\text{6 marks}}$ Use the **definition** of the derivative to show that

$$f(x) = \begin{cases} 2x+2\,, & x \leq 1 \\ 4\sqrt{x}\,, & x > 1 \end{cases}$$

is differentiable at $x = 1$.

(b) $\boxed{\text{4 marks}}$ Use the properties of logarithms to simplify, and then find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ if

$$y = \ln\left(\sqrt{\frac{\sin x}{1-2x}}\right)$$

3. (a) $\boxed{\text{7 marks}}$ Given that the function $f(x) = x^5 + 3x^3 + x$ has the inverse $g(x)$, find

(i) $g'(5)$ and (ii) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[f\left(2g(x)\right)\right]$ at $x = 5$.

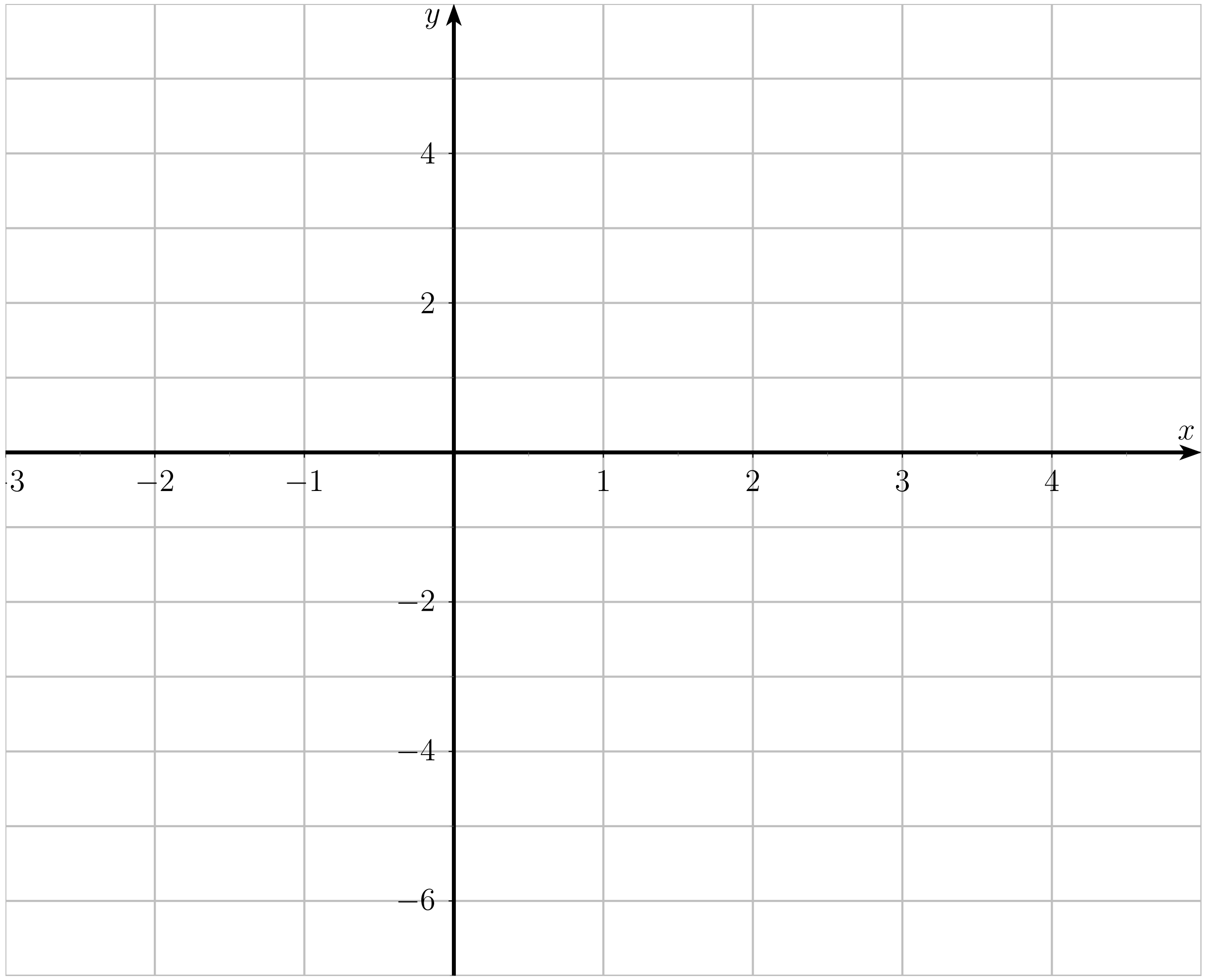
(b) $\boxed{\text{4 marks}}$ Use the Mean Value Theorem to show that $|\tan^{-1} a| < |a|$ for all $a \neq 0$.

4. $\boxed{\text{10 marks}}$ An open box is to be made from a 24 cm by 24 cm square piece of cardboard by cutting out squares of equal size from the four corners and folding up the sides. What should be the dimensions of the squares to obtain a box with the largest volume?

5. $\boxed{\text{15 marks}}$ Given $f(x) = \dfrac{1}{(x-1)(x-2)}$, $f'(x) = \dfrac{3-2x}{(x-1)^2(x-2)^2}$, and

$$f''(x) = \frac{2\,(3x^2-9x+7)}{(x-1)^3(x-2)^3},$$ find:

(a) x and y intercepts, **(b)** vertical asymptotes and the behaviour of f near the vertical asymptotes, **(c)** horizontal asymptotes, **(d)** critical numbers, **(e)** intervals in which f increases and decreases, **(f)** local extrema, **(g)** concavity and the $x-$coordinates of any inflection points. Then sketch the graph of f in the **next page # 7**.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6. $\boxed{\text{7 marks}}$ Use the limit of Riemann sum to compute the area under the curve $y = 16 - x^2$ over the interval $[0,\ 4]$.

7. $\boxed{4+5 \text{ marks}}$ Evaluate the following integrals using suitable substitutions:

(a) $\displaystyle\int \frac{9}{x(2+3\ln x)^4}\,\mathrm{d}x$

(b) $\displaystyle\int_2^5 \frac{x-2}{\sqrt{x-1}}\,\mathrm{d}x$

8. (a) $\boxed{\text{6 marks}}$ Show that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$ for all x.

(b) $\boxed{\text{3 marks}}$ Given $y = x \sinh x$, find $y''(0)$.

The remainder of this exam consists of **Multiple Choice** questions. Circle the correct answer for each question. **No partial credit will be given.** (2 marks for each question)

9. The exact value of $\tanh(\ln 3)$ is

(A) $\frac{5}{4}$ **(B)** 0 **(C)** $\frac{4}{5}$ **(D)** none of them

10. Let f be a differentiable function of x, and $g(x) = f(b + mx) + f(b - mx)$, where b and m are non-zero constants. Then $g'(0)$ is

(A) 1 **(B)** 0 **(C)** b **(D)** m

11. If $v(t) = 2\sin t$ is the velocity of a particle at time t, then the average value of v on $0 \leq t \leq \frac{\pi}{2}$ is

(A) $\frac{4}{\pi}$ **(B)** $\frac{1}{\pi}$ **(C)** 2 **(D)** none of them

12. $\displaystyle\lim_{x\to 0^+} \frac{\ln x}{x-1-e^x}$ is

(A) $-\infty$ **(B)** 0 **(C)** ∞ **(D)** none of them

13. If the function $f(x)$ has critical numbers at $x=-1$, $x=0$, $x=1$ and if $f''(-1)<0$, $f''(0)=0$, $f''(1)>0$, then the graph of $y=f(x)$ has a local minimum at

(A) $x=0$ **(B)** $x=-1$ and $x=1$ **(C)** $x=-1$ **(D)** $x=1$

14. $\displaystyle\lim_{x\to 1} \frac{\int_1^x \sqrt{t^5+8}\,\mathrm{d}t}{x-1}$ is

(A) 0 **(B)** 3 **(C)** 1 **(D)** $2\sqrt{2}$

15. The graph of $y=2x+x^{4/3}$ is concave up in the interval

(A) $(-\infty,\ 0)$ **(B)** $(-\infty,\ 0)\cup(0,\ \infty)$ **(C)** $(0,\ \infty)$ **(D)** none of them

16. If $g(x) = x\ln(-x)$, then $g'(-e)$ is

(A) 0 **(B)** $1 - \frac{1}{e}$ **(C)** 2 **(D)** none of them

17. If $f(x) = 7 + g(x)$ for $3 \leq x \leq 5$, and $\int_3^5 g(x)\,\mathrm{d}x = -4$ then $\int_3^5 [f(x) + g(x)]\,\mathrm{d}x$ is

(A) -1 **(B)** 6 **(C)** 10 **(D)** 3

18. $\sum_{i=0}^{30} (3 + i)$ is

(A) 555 **(B)** 558 **(C)** 468 **(D)** none of them

THE END OF THE EXAM

This page is for rough work. It will not be graded.

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