MATH 2107 – CALCULUS I (FALL 2010)
MODEL SOLUTIONS (TEST 1)

① (a) $$\lim_{h\to 2} \frac{\sqrt{h}-\sqrt{2}}{h^2+h-6}$$

$$= \lim_{h\to 2} \frac{\sqrt{h}-\sqrt{2}}{(h^2+h-6)} \cdot \frac{\sqrt{h}+\sqrt{2}}{\sqrt{h}+\sqrt{2}}$$

$$= \lim_{h\to 2} \frac{(h-2)}{(h-2)(h+3)(\sqrt{h}+\sqrt{2})}$$

$$= \lim_{h\to 2} \frac{1}{(h+3)(\sqrt{h}+\sqrt{2})} = \frac{1}{10\sqrt{2}}$$

(b) $$\lim_{x\to -\infty} \frac{x}{\sqrt{9x^2-2x+5}}$$

$$= \lim_{x\to -\infty} \frac{x}{|x|\sqrt{9-\frac{2}{x}+\frac{5}{x^2}}}$$

$$= \lim_{x\to -\infty} \frac{x}{-x\sqrt{9-\frac{2}{x}+\frac{5}{x^2}}} = -\frac{1}{\sqrt{9-0+0}} = -\frac{1}{3}$$

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② (a) For f to be continuous at $x=0$, we need

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

or $$\lim_{x\to 0^-} (e^{|x|}+K) = \lim_{x\to 0^+} (4-x^2) = 4$$

$$\Rightarrow \quad 1+K = 4 = 4$$

$$\Rightarrow \quad K = 3$$

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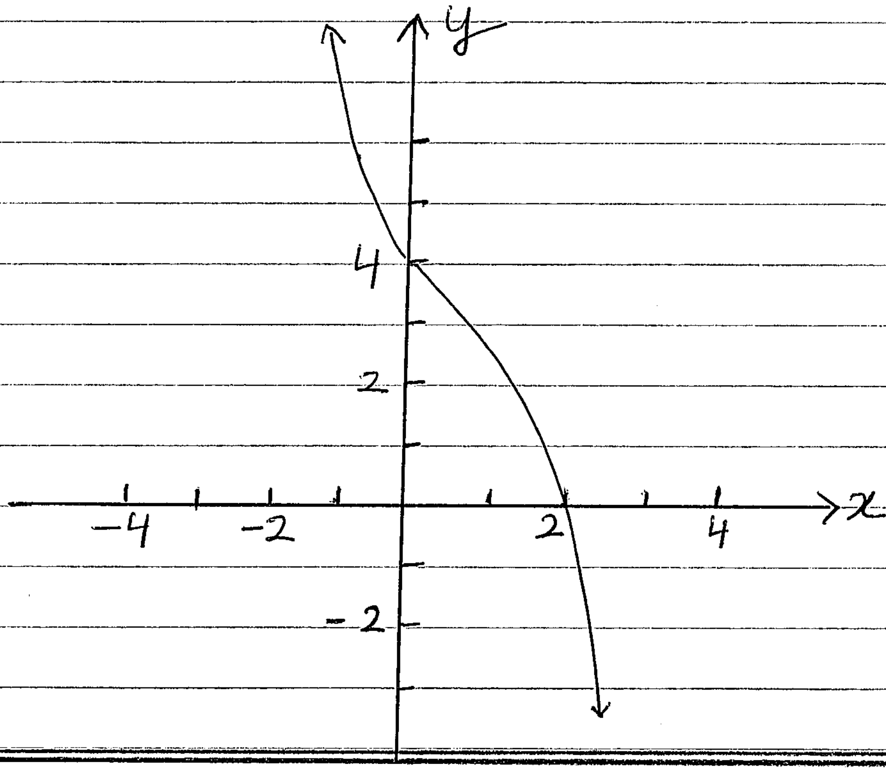
② (b) $f(x) = \begin{cases} e^{-x} + 3, & x < 0 \\ 4 - x^2, & x \geq 0 \end{cases}$



③ f is continuous in $[0,3]$

$f(0) = 1 - 1 - 2 = -2 < 0$
$f(3) = e^6 - e^3 - 2 > 0$
$\therefore$ $f(0)$ and $f(3)$ are of opposite signs

Thus by the Intermediate Value Theorem, there is a number c in $(0,3)$ such that $f(c) = 0$.

$f(c) = 0 \Rightarrow e^{2c} - e^c - 2 = 0$

$\Rightarrow (e^c - 2)(e^c + 1) = 0$

$\Rightarrow e^c = 2 \quad (\because e^c \neq -1)$

$\Rightarrow \boxed{c = \ln 2}$

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(4) $g'(x) = \lim_{h \to 0} \dfrac{g(x+h) - g(x)}{h}$

$$= \lim_{h \to 0} \frac{\dfrac{3}{1+2x+2h} - \dfrac{3}{1+2x}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{3+6x-3-6x-6h}{(1+2x+2h)(1+2x)}}{h}$$

$$= \lim_{h \to 0} \frac{-6\cancel{h}}{\cancel{h}(1+2x+2h)(1+2x)}$$

$$= -\frac{6}{(1+2x)(1+2x)} = -\frac{6}{(1+2x)^2}$$

(5) $y = x^2 (2-x^2)^{1/3}$

$$y' = 2x(2-x^2)^{1/3} + x^2\left[\frac{1}{3}(2-x^2)^{-2/3}(-2x)\right]$$

$\therefore$ $m = \text{slope} = y'(1) = (2)(1) + (1)\left[\frac{1}{3}(1)(-2)\right]$

$$= 2 - \frac{2}{3} = \frac{4}{3}$$

$y(1) = (1)(1) = 1$

$\therefore$ Equation of the tangent line is

$$y - 1 = \frac{4}{3}(x-1)$$

$$\left[\text{or } \; y = \frac{4}{3}x - \frac{1}{3}\right]$$

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⑥ (a) $S = \left(\dfrac{t^2-3t}{t+1}\right)^2$

$$V(t) = S'(t) = 2\left(\frac{t^2-3t}{t+1}\right)\frac{d}{dt}\left(\frac{t^2-3t}{t+1}\right)$$

$$= 2\left(\frac{t^2-3t}{t+1}\right)\left[\frac{(2t-3)(t+1)-(t^2-3t)}{(t+1)^2}\right]$$

(b)

$$V(t) = 2\left(\frac{t^2-3t}{t+1}\right)\left(\frac{2t^2-t-3-t^2+3t}{(t+1)^2}\right)$$

$$= 2\,\frac{(t^2-3t)(t^2+2t-3)}{(t+1)^3}$$

$$V(t)=0 \Rightarrow t(t-3)(t+3)(t-1)=0$$

$$\Rightarrow \boxed{t = 0, 3 \text{ or } 1} \quad (\because t \neq -3)$$

MULTIPLE CHOICE QUESTIONS (KEYS)

VERSION I		VERSION II	
Q. No.	CORRECT CHOICE	Q. NO	CORRECT CHOICE
7	(C)	7	(B)
8	(A)	8	(D)
9	(B)	9	(C)
10	(B)	10	(C)
11	(B)	11	(B)