Sultan Qaboos University
College of Science, Department of Mathematics and Statistics
MATH 3171-Linear Algebra and Multivariate Calculus for Engineers
Spring 2010, Homework

To be submitted on 12 April 2010 during the class time and will be assessed by a Quiz

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Solve the following 10 problems in details. No photocopy or a typed version will be accepted

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1. Let A be the matrix $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$. Find $P(A)$ if $P(x) = \lambda x^2 - x + 1$ where λ is the first digit of your student ID.

2. For which values of a will the following system have no solutions? Exactly one solution? Infinitely many solutions?
$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2. \end{aligned}$$

3. Consider the matrix $A = \begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{bmatrix}$.
   (a) Find $\det(A)$ and $\operatorname{rank}(A)$.
   (b) What can be said about the linear dependence and independence of the row vectors $\begin{bmatrix} 3 & 5 & -2 & 6 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 & -1 & 1 \end{bmatrix}$, $\begin{bmatrix} 2 & 4 & 1 & 5 \end{bmatrix}$ and $\begin{bmatrix} 3 & 7 & 5 & 3 \end{bmatrix}$? Justify your answer.

4. Solve the following system for x, y, and z by using Cramer's rule;
$$\begin{aligned} \frac{1}{x} + \frac{2}{y} - \frac{4}{z} &= \lambda \\ \frac{2}{x} + \frac{3}{y} + \frac{8}{z} &= 0 \\ -\frac{1}{x} + \frac{9}{y} + \frac{10}{z} &= 5 \end{aligned}$$
   where λ is the first digit of your student ID.

5. Express the following system as $A\mathbf{X} = \mathbf{b}$. Hence solve it by calculating the inverse of the coefficient matrix
$$\begin{aligned} x \qquad + 3z &= \lambda \\ 3x + 4y - z &= 3 \\ 2x + 5y - 4z &= 1 \end{aligned}$$
   where λ is the first digit of your student ID.

6. Find all the values of a, b, and c for which A is skew-symmetric

$$A = \begin{bmatrix} 0 & a-2b+2c & 2a+b+c \\ 3 & 0 & a+c \\ 0 & -2 & 0 \end{bmatrix}.$$

7. Find a nonsingular matrix P that reduces $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ to a diagonal matrix D.

   Verify your results by calculating $PD = AP$.

8. Let $A(1,1,1)$, $B(0,2,2)$, $C(3,11,3)$, and $D(3,3,10)$.

   (i) Find the angles (in degrees) of the triangle with vertices A, B and C.

   (ii) Find an equation of the plane passing through the points A, B and C.

   (iii) Find the volume of the parallelepiped with vertices A, B, C and D.

9. (i) Let $\mathbf{a} = [3,1,-2]$, $\mathbf{b} = [-5,7,0]$, and $\mathbf{c} = [4,-6,0]$. Find $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$, $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$.

   (ii) Let $\mathbf{u}(t) = [u_1(t), u_2(t), u_3(t)]$, $\mathbf{v} = [v_1(t), v_2(t), v_3(t)]$, and $\mathbf{w} = [w_1(t), w_2(t), w_3(t)]$.
   Prove that $(\mathbf{u}\ \ \mathbf{v}\ \ \mathbf{w})' = (\mathbf{u}'\ \ \mathbf{v}\ \ \mathbf{w}) + (\mathbf{u}\ \ \mathbf{v}'\ \ \mathbf{w}) + (\mathbf{u}\ \ \mathbf{v}\ \ \mathbf{w}')$ where prime denotes differentiation w. r. to t.

   (iii) Let $f(x,y,z) = \ln(x^3y^4z^2 + xyz) - \sin(xyz) + e^{x-y-z}$. Find $\dfrac{\partial f}{\partial x}(1,1,1)$, $\dfrac{\partial f}{\partial y}(1,1,1)$, $\dfrac{\partial f}{\partial z}(1,1,1)$.

10. (i) Find the tangent vector and the equation of the tangent line to the curve

    $$\mathbf{r}(t) = \lambda\cos t\ \mathbf{i} - \lambda\sin t\ \mathbf{j} + 4t\ \mathbf{k} \text{ at } P:(0,-\lambda,2\pi)$$

    where λ is the first digit of your student ID.

    (ii) Find the length of the cycloid $\mathbf{r} = \lambda[t - \sin t,\ 1-\cos t]$ from $t=0$ to $t = 2\pi$
    where λ is the first digit of your student ID.