SULTAN QABOOS UNIVERSITY – COLLEGE OF SCIENCE
DEPARTMENT OF MATHEMATICS AND STATISTICS
SPRING- 2010 FINAL EXAMINATION
MATH 3171 – LINEAR ALGEBRA AND MULTIVARIATE CALCULUS FOR ENGINEERS

TIME ALLOWED: 150 MINUTES **TOTAL MARKS: 80**

SHOW ALL WORK FOR FULL CREDIT

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1. Consider the system of equations
$$\begin{aligned} x + y + 2z &= \alpha \\ x \quad\;\; + z &= \beta \\ 2x + y + 3z &= \gamma \end{aligned}.$$

(a) **[7 Marks]** Reduce the augmented matrix of the given system to the echelon form and:

(i) determine a condition on the constants α, β, and γ for which solution of the system exits;

(ii) hence find the solution of the given system .

(b) **[1 Mark]** Find the rank of the coefficient matrix of the given system.

(c) **[2 Marks]** Determine whether the row vectors $\begin{bmatrix}1 & 1 & 2\end{bmatrix}$, $\begin{bmatrix}1 & 0 & 1\end{bmatrix}$ and $\begin{bmatrix}2 & 1 & 3\end{bmatrix}$ of the coefficient matrix are linearly independent or not. Justify your answer.

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2. (a) **[3 Marks]** Find $\det(A)$ if $(I + 2A)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$.

(b) **[8 Marks]** Given a matrix $B = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & -2 \\ 6 & -3 & 1 \end{bmatrix}$.

(i) Find the inverse of B, if it exists.

(ii) Hence find X if $BX = D$ where $D = \begin{bmatrix} 16 \\ 0 \\ 16 \end{bmatrix}$.

(c) **[3 Marks]** Prove that the determinant of a unitary matrix has absolute value 1.

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3. (a) **[2 Marks]** Prove that if X is an eigenvector for both of the matrices A and B, then X is also an eigenvector of the matrix $A + B$.

(b) **[7 Marks]** Find the eigenvalues and the corresponding eigenvectors of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$.

(c) **[3 Marks]** Diagonalize A if possible.

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4. (a) **[3 Marks]** Find $\frac{\partial w}{\partial u}$ if $w = \frac{1}{8}(x^2 + y^2 + z^2)$, $x = u^2 + v^2$, $y = u^2 - v^2$, and $z = 2uv$. Write your answer in terms of u and v.

(b) **[3 Marks]** Given $f(x, y, z) = 3x^2 - 4y^2 + z^2 - 12$.

(i) At $P:(2,1,2)$ along what direction will the change of f be maximum?

(ii) Find the maximum change in f at $P:(2,1,2)$.

(c) **[2 Marks]** Consider the vector field $\mathbf{F}(x, y, z) = [x^2yz, -xy^2z, 5x + z]$. Explain why $\mathbf{F}$ is not the curl of another vetor field $\mathbf{G}$.

(d) **[5 Marks]** Let $f(x, y, z) = x - z$ and $\mathbf{v}(x, y, z) = \left[y, z, 2x\right]$, verify that $\text{curl}(f\ \mathbf{v}) = \nabla f \times \mathbf{v} + f\ \text{curl}(\mathbf{v})$.

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5. (a) **[3 Marks]** Find the arc length of the curve $\mathbf{r}(t) = a\cos^3 t\ \mathbf{i} + a\sin^3 t\ \mathbf{j}$, $0 \le t \le \frac{\pi}{2}$.

(b) **[6 Marks]** Find a potential of the vector field $\mathbf{F} = \left[2xy + \cos x,\ x^2, 1\right]$ if it exists.

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6. (a) **[3 Marks]** If $\mathbf{r}(u, v) = \cos u \cos v\ \mathbf{i} + (2 + \sin u \cos v)\ \mathbf{j} + (1 + \sin v)\ \mathbf{k}$; $0 \le u \le 2\pi$, $-\frac{\pi}{2} \le v \le \frac{\pi}{2}$ represents some surface parametrically then find the equation of this surface in the cartesian coordinate system.

(b) **[7 Marks]** Compute the flux $\iint_S \mathbf{F}.\mathbf{n}\ dA$ of water through the first octant of the surface $S: x + 2y + z = 4$ if the velocity vector is $\mathbf{v} = \mathbf{F} = [y, z, x]$, speed being measured in meter/sec. (Density of water is 1 ton/meter^3).

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7. (a) **[6 Marks]** Evaluate the line integral $\int_C \mathbf{F}.\mathbf{r}'(s)\ ds$ by Stokes's theorem where $\mathbf{F} = [3y, 8x, 3y]$ and C is the circle: $x^2 + y^2 = 16$, $z = 4$, oriented counterclockwise.

(b) **[6 Marks]** Use Gauss divergence theorem to evaluate $\iint_S \mathbf{F}.\mathbf{n}\ dA$ for the vector field $\mathbf{F} = [2x^3, -2x^2y, x^2z]$ and the surface $S:\ x^2 + y^2 = 4$, $|z| \le 2$.

TOTAL 80 MARKS ***WISH YOU GOOD LUCK***