Sultan Qaboos University
College of Science, Department of Mathematics and Statistics
MATH 3171-Linear Algebra and Multivariate Calculus for Engineers
Fall 2010, Homework

To be submitted on 20 November 2010 during the class time and will be assessed by a Quiz

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Solve the following 10 problems in details. No photocopy or a typed version will be accepted

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1. The curve $y = ax^2 + bx + c$ passes through the points $(x_1, \ y_1)$, $(x_2, \ y_2)$ and $(x_3, \ y_3)$. Show that the coefficients a, b, and c are a solution of the system of linear equations whose augmented matrix is $\begin{bmatrix} x_1^2 & x_1 & 1 & y_1 \\ x_2^2 & x_2 & 1 & y_2 \\ x_3^2 & x_3 & 1 & y_3 \end{bmatrix}$.

2. Solve the following system of nonlinear equations for the unknown angles α, β, and γ, where $0 \le \alpha \le 2\pi$, $0 \le \beta \le 2\pi$, and $0 \le \gamma \le 2\pi$.
$$2\sin\alpha - \cos\beta + 3\tan\gamma = 3$$
$$4\sin\alpha + 2\cos\beta - 2\tan\gamma = 2$$
$$6\sin\alpha - 3\cos\beta + \tan\gamma = 9.$$

3. Consider the matrix $A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$.

   (i) Find $\operatorname{rank}(A)$.

   (ii) With proper explanation determine whether the column vectors of the matrix A are linearly dependent or independent.

4. Find A if $\left(3A^T\right)^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$. Hence solve for X if $AX = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$.

5. Determine whether the matrix $A=\begin{bmatrix} \frac{1+i}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{i}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{i}{\sqrt{3}} \\ \frac{3+i}{2\sqrt{15}} & \frac{4+3i}{2\sqrt{15}} & \frac{5i}{2\sqrt{15}} \end{bmatrix}$ is Hermitian, Skew-Hermitian or Unitrary.

6. Find a nonsingular matrix X that reduces $A=\begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$ to a diagonal matrix D. Verify your results by calculating $XD = AX$.

7. Let $A\left(-1,1,-1\right)$, $B\left(2,0,2\right)$, $C\left(4,1,-3\right)$, and $D\left(-3,1,10\right)$.

   (i) Find the angles (in degrees) of the triangle with vertices A, B and C.

   (ii) Find an equation of the plane passing through the points A, B and C.

   (iii) Find the volume of the tetrahedron if the vertices are A, B, C and D.

8. Let $f(x,y,z)=\cos(x^2yz)+e^{x+y-z}-\ln(xyz)$.

   (i) Find $\frac{\partial f}{\partial x}(1,1,1)$, $\frac{\partial f}{\partial y}(1,1,1)$, $\frac{\partial f}{\partial z}(1,1,1)$.

   (ii) Find the directional derivative of $f(x,y,z)=\cos(x^2yz)+e^{x+y-z}-\ln(xyz)$ at $P:(1,1,1)$ in the direction of $\mathbf{a}=i-2k$.

9. (i) Find the tangent vector and the equation of the tangent line to the curve $\mathbf{r}(t)=-a\cos t\ \mathbf{i}+a\sin t\ \mathbf{j}+5t\ \mathbf{k}$ at $P:(a,0,5\pi)$.

   (ii) Find the length of the curve $\mathbf{r}=\left[t,\quad t^2,\quad t^3\right]$ from $t=0$ to $t=\pi$.

10. (i) Show that the vector field $\mathbf{F}=\left[2xy,\quad x^2+2y,\quad -3z^2\right]$ is conservative. Hence find a potential of the vector field $\mathbf{F}$.

    (ii) Let $f=f(x,y,z)$ and $g=g(x,y,z)$. Assuming sufficient differentiability prove that $div(f\nabla g)-div(g\nabla f)=f\nabla^2 g-g\nabla^2 f$.