SULTAN QABOOS UNIVERSITY – COLLEGE OF SCIENCE
DEPARTMENT OF MATHEMATICS AND STATISTICS
SUMMER 2010 – INTERM TEST

MATH 3171 – LINEAR ALGEBRA AND MULTIVARIATE CALCULUS FOR ENGINEERS

TIME ALLOWED: 70 MINUTES

SECTION: **ID. NO.** **NAME:**

INSTRUCTIONS

- **SHOW ALL WORK FOR FULL CREDIT.**
- **THE TEST CONSISTS OF 4 QUESTIONS, EACH ON A SEPARATE SHEET.**
- **PROVIDE YOUR ANSWERS ON THE SHEET IN THE SPACE DIRECTLY BELOW EACH QUESTION.**
- **PLEASE DO NOT SEPARATE THE PAGES OF THIS BOOKLET**

WRITE YOUR ID AT THE TOP OF EACH SHEET

QUESTION NO.	MAXIMUM MARKS	MARKS SCORED
1	15	
2	15	
3	15	
4	15	
TOTAL	60	

1. (a) [10 Marks] Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$.

(i) **Show that** A ***satisfies*** **the expression** $A^3 - 2A^2 + A - I = \mathbf{0}$ **.**

(ii) **Show that** A **is *invertible*. Find the inverse of** A ***using the above expression*.**

(b) [5 Marks] Show that any square matrix can be written as the sum of a *symmetric* matrix and a *skew- symmetric* matrix.

2. Consider the non-homogeneous linear system

$$\begin{aligned} w \quad -x \quad +y \quad +2z &= 2 \\ 3w \quad +x \quad +4y \quad -z &= 4 \\ 11w \quad +x \quad +14y \quad +z &= \lambda. \end{aligned}$$

(a) [4 Marks] Reduce the *augmented* matrix to the echelon form.

(b) [7 Marks] Determine the value(s) of λ for which this system has
(i) no solution,
(ii) at least one solution. For case (ii) write the *general solution* in matrix form.

(c) [2 Mark] Using part (a) find the rank of the *augmented* matrix.

(d) [2 Mark] Using part (a) determine whether the row vectors $\begin{bmatrix} 1 & -1 & 1 & 2 \end{bmatrix}$, $\begin{bmatrix} 3 & 1 & 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 11 & 1 & 14 & 1 \end{bmatrix}$ are linearly *dependent* or *independent*. Justify your answer.

3. (a) [7 Marks] Solve the following system by Cramer's rule *if it is applicable*

$$\begin{aligned} x + y \quad\quad &= 2 \\ y + z &= 6 \\ x \quad\quad + z &= 4. \end{aligned}$$

(b) [4 Marks] If A **,** B **and** C **are nonsingular square matrices of any order, then *prove* that** $(\mathrm{ABC})^{-1} = C^{-1}\mathrm{B}^{-1}A^{-1}$ **.**

(c) [4 Marks] Prove that the product of any two unitary matrices (of the same size) is unitary.

4. Consider $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ **.**

(a) [7 Marks] Find the *eigenvalues* and the corresponding *eigenvectors* of A **.**

(b) [3 Marks] Find *algebraic* multiplicity, *geometric* multiplicity and *defect* of the corresponding eigenvalues of the matrix A **.**

(c) [5 Marks] With proper explanation determine whether A **is diagonalizable nor not. If possible diagonalize the matrix** A **.**