Date: 17 September 2007   **Time Allowed: 20 minutes**
Name: **ID NO.:**

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1. Let $u = \begin{bmatrix} a & b & c \end{bmatrix}$, $v = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Compute $A^T u^T$ and $u^T v v^T$.

2. Solve the following linear system by Gauss elimination:

$$\begin{cases} x_1 & - & x_2 & + & 2x_3 & - & x_4 & = & 0 \\ -3x1 & + & 3x_2 & - & 6x_3 & + & 5x_4 & = & 12 \\ 2x_1 & - & 2x_2 & + & 4x_3 & - & 2x_4 & = & 0 \end{cases}.$$

3. Find the rank of $A = \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 0 & 3 \end{bmatrix}$.

Answers:

Date: 01 October 2007   **Time Allowed: 20 minutes**
Name: **ID NO.:**

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1. [10 marks] Let $A = \begin{bmatrix} -3 & 2 & 3 \\ 0 & -1 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 2 & 3 \\ 0 & -3 & 5 \\ 0 & 0 & -4 \end{bmatrix}$.

(i)[5 marks] Evaluate the determinant of the matrix $2(A - B)^T$.

(ii)[3 marks] Use the determinant concept to evaluate the rank of the matrix $(A - B)^5$.

(iii)[2 marks] Does the inverse of the matrix $(A - B)^5$ exists ? Justify your answer.

2. [10 marks] Given the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 5 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$.

(i)[4 marks] Find the eigenvalues of A.

(ii)[6 marks] Find an eigenvector corresponding to the smallest eigenvalue of A.

Answers:

Date: 5 November 2007   **Time Allowed: 20 minutes**
Name: **ID NO.:**

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1. Let $\mathbf{a} = [1, -2, 1], \mathbf{b} = [-1, -2, 1], \mathbf{c} = [1, 1, 2]$. Find

$$(\mathbf{c}.\mathbf{a})\mathbf{b} - (\mathbf{c}.\mathbf{b})\mathbf{a} .$$

2. Find two vectors perpendicular to both

$$\mathbf{u} = [1, -2, 1], \text{ and } \mathbf{v} = [3, 1, -2].$$

3. Find the moment vector about the point $(3, -5, 4)$ of the force $\mathbf{p} = [1, -3, 2]$ acting at the point $(1, 2, 1)$.

Answers:

Date: 19 November 2007   **Time Allowed: 20 minutes**
Name: **ID NO.:**

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1. Consider the surface $S : e^{xy+z} = 1$.
   Find a unit vector normal to the surface S at the point $P : (1, -1, 1)$.

2. Find the length of the curve $r(t) = [2\cos t, 2\sin t, 6t]$ from $A : (2, 0, 0)$ to
   $B : (2, 0, 12\pi)$.

3. Let $f(x, y, z)$ be differentiable scalar function and $\mathbf{v}(x, y, z)$ be differentiable vector
   function. Prove that: $\text{div}(f\mathbf{v}) = f\text{div}(\mathbf{v}) + \mathbf{v} \cdot \nabla f$.

Answers:

Date: 3 December 2007 **Time Allowed: 20 minutes**
Name: **ID NO.:**

1. [7 marks] Let $f = xy - yz$, $\mathbf{v} = [2y, z, 2x]$. Find $\operatorname{curl}(f\mathbf{v})$.

2. [7 marks] Evaluate the integral $\int_C \sqrt{2 + x^2 + 3y^2}\, ds$; where
   $C: r(t) = [-2t, -2t, 4t^2], -1 \le t \le 0$.

3. [6 marks] Let $I = \int_{(1,1,-1)}^{(2,0,2)} (2xy + z^2)\,dx + (x^2 - 2yz)\,dy + (2xz - y^2)\,dz$. Show that the
   integral is path independent and evaluate I.

Answers: